Name: Key

Show your work. Answers without work earn reduced credit.

1. **[3 parts, 1 point each]** An economist is interested in how the price of a certain item affects its sales. At a price of p dollars, a quantity $q = f(p)$ of the item is sold.

   (a) Interpret in English: $f(120) = 7200$.

   When the price is \$120, the quantity of units that sell is 7,200.

   (b) Interpret in English: $f'(120) = -50$.

   When the price is \$120, increasing the price has the effect of reducing the number of items sold by 50 items per dollar.

   (c) Using the given information, estimate $f(118)$.

   $$f(118) \approx f(120) + m \cdot \Delta x$$
   $$= 7200 + (-50)(-2) = \boxed{7,300 \text{ items}}.$$

2. **[3 parts, 1 point each]** Let $P(t)$ be the world's population (in billions of people) t years after 2000.

   (a) Translate the following two sentences into mathematical equations. In 2005, the world's population was 6.51 billion. In 2010, the world's population was 6.91 billion.

   $P(5) = 6.51$

   $P(10) = 6.91$

   (b) Based on the information given, estimate $P'(5)$. Include units.

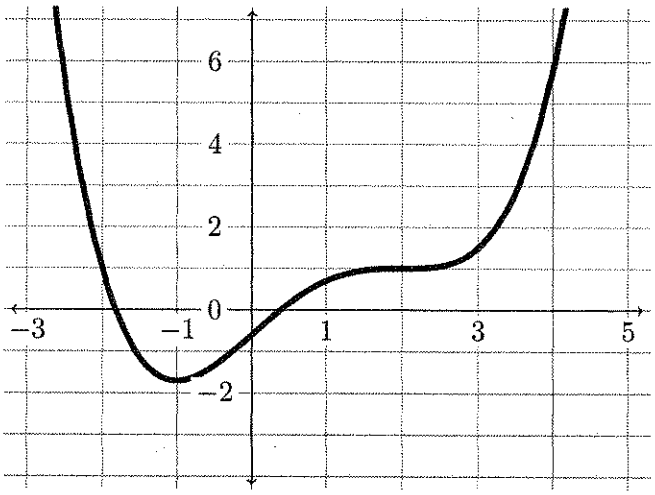
   $$P'(5) \approx \frac{P(10) - P(5)}{10 - 5} = \frac{6.91 - 6.51}{5} = \frac{0.4}{5} = \boxed{0.08 \text{ billion people per year}}$$

   (c) Compute the relative rate of change in the world's population at $t = 5$. Include units and explain what this value means in terms of the world's population.

   $$RROC = \frac{P'(5)}{P(5)} = \frac{0.08}{6.51} \approx \boxed{0.0123 \text{ per year}}.$$

   This means that in 2005, the world population was increasing by 1.23% per year.

3. [**2 parts, 2 points each**] The graph of a function $f(x)$ appears below.



(a) Estimate the intervals on which the derivative of f is positive and the intervals on which the derivative of f is negative.

f' negative: $(-\infty, -1)$

f' positive: $(-1, 2)$ and $(2, \infty)$

(the answer $(-1, \infty)$ is also OK)

(b) Estimate the intervals on which the second derivative of f is positive and the intervals on which the second derivative of f is negative.

f'' negative: $(0.25, 2)$

(Also OK: $(0, 2)$ or $(1, 2)$.)

f'' positive: $(-\infty, 0.25)$ and $(2, \infty)$.