

Name: Key

Show your work. Answers without work earn reduced credit.

1. [2 parts, 1 point each] An air-freshener starts with 50 grams and evaporates. In each of the following cases, write a formula for the quantity  $Q$  in grams of air-freshener remaining  $t$  days after the start. The decrease is:

- (a) 3 grams a day

$$Q = 50 - 3t$$

- (b) 16% a day

$$Q = 50(1 - 0.16)^t$$

$$Q = 50(0.84)^t$$

2. [4 parts, 1 point each] Solve the following equations for  $t$  exactly. Decimal approximations are worth partial credit.

- (a)  $3^t = 4$

$$\ln(3^t) = \ln(4)$$

$$t \ln(3) = \ln(4)$$

$$t = \frac{\ln(4)}{\ln(3)}$$

- (b)  $4e^{2t} = 12$

$$\ln(4e^{2t}) = \ln(12)$$

$$e^{2t} = 3$$

$$\ln(e^{2t}) = \ln(3)$$

$$2t = \ln(3)$$

$$t = \frac{\ln(3)}{2}$$

- (c)  $7\left(\frac{2}{3}\right)^t = 2$

$$\left(\frac{2}{3}\right)^t = \frac{2}{7}$$

$$\ln\left(\left(\frac{2}{3}\right)^t\right) = \ln\left(\frac{2}{7}\right)$$

$$t \cdot \ln\left(\frac{2}{3}\right) = \ln\left(\frac{2}{7}\right)$$

$$t = \frac{\ln\left(\frac{2}{7}\right)}{\ln\left(\frac{2}{3}\right)}$$

- (d)  $6e^{-t} = 2^t$

$$\ln(6 \cdot e^{-t}) = \ln(2^t)$$

$$\ln(6) + \ln(e^{-t}) = t \cdot \ln(2)$$

$$\ln(6) - t = t \cdot \ln(2)$$

$$\ln(6) = t + t \cdot \ln(2)$$

$$\ln(6) = t(1 + \ln(2))$$

$$t = \frac{\ln(6)}{1 + \ln(2)}$$

3. [2 points] Find the half-life of a quantity that decreases at a discrete rate of 6% each month.

$$P = P_0 (1 - 0.06)^t$$

$$\frac{1}{2} = 1 \cdot (0.94)^t$$

$$\ln\left(\frac{1}{2}\right) = \ln(0.94)^t$$

$$\ln\left(\frac{1}{2}\right) = t \cdot \ln(0.94)$$

$$t = \frac{\ln\left(\frac{1}{2}\right)}{\ln(0.94)} \text{ months}$$

or

$$t \approx 11.202 \text{ months}$$

4. [2 parts, 1 point each] You are negotiating a contract with a client, and three versions are proposed. Contract A calls for the client to make three payments of \$1000 each, to be paid now, one year from now, and two years from now. Contract B calls for a single payment of \$3200 to be paid in two years. Contract C requires a single payment of \$2900 now. You estimate that invested money will grow at a continuous rate of 5% each year.

- (a) Find the future value of all three contracts in 2 years.

Contract A:  $P = 1000 e^{0.05(2)} + 1000 e^{0.05(1)} + 1000$

$$\approx \$3156.44$$

Contract B:  $P = \$3200$

Contract C:  $P = 2900 e^{0.05(2)} \approx \$3205.00$

- (b) Find the present value of all three contracts.

Contract A:  $P_0 = \frac{3156.44}{e^{0.05(2)}}$

$$= \$2856.07$$

Contract B:  $P_0 = \frac{3200}{e^{0.05(2)}} =$

$$\$2895.48$$

Contract C:  $P_0 = \$2900$

$$P = P_0 e^{0.05t}$$

$$P_0 = \frac{P}{e^{0.05t}}$$