Name: Key

Show your work. Answers without work earn reduced or no credit.

1. **[2 points]** Find an antiderivative for the following functions.

(a) $f(x) = 3$

$F(x) = 3x$

(b) $f(t) = 4t - 2$

$F(t) = 2t^2 - 2t$

(c) $g(x) = x^{\ln(3)}$

$G(x) = \frac{1}{\ln(3)+1} x^{\ln(3)+1}$

(d) $h(y) = y^2 + \frac{1}{y}$

$H(y) = \frac{y^3}{3} + \ln|y|$

2. **[2 points]** Find the following indefinite integrals.

(a) $\int (t^2+1)^2 \, dt$

$= \int t^4 + 2t^2 + 1 \, dt$

$= \boxed{\frac{1}{5}t^5 + \frac{2}{3}t^3 + t + C}$

(b) $\int \left(3x - \frac{1}{\sqrt{x}}\right) dx$

$= \frac{3}{2}x^2 - \int x^{-\frac{1}{2}} dx$

$= \frac{3}{2}x^2 - 2x^{\frac{1}{2}} + C$

$= \boxed{\frac{3}{2}x^2 - 2\sqrt{x} + C}$

(c) $\int 2e^{-r} \, dr$

$= \boxed{-2e^{-r} + C}$

(d) $\int (e^{3y} - 1) \, dy$

$= \boxed{\frac{1}{3}e^{3y} - y + C}$

3. **[2 parts, 2 points each]** Find the following indefinite integrals.

(a) $\int 2x(x^2+1)\,dx$ $= \int \cancel{2x} \cdot w \cdot \frac{1}{\cancel{2x}}\,dw$

$$\boxed{\begin{aligned} w &= x^2+1 \\ \frac{dw}{dx} &= 2x \\ \frac{1}{2x}\,dw &= dx \end{aligned}}$$

$= \int w\,dw$

$= \frac{w^2}{2} + C = \boxed{\frac{(x^2+1)^2}{2} + C}$

(b) $\int te^{3t^2}\,dt$ $= \int \cancel{t} e^w \cdot \frac{1}{6\cancel{t}}\,dw$

$w = 3t^2$

$\frac{dw}{dt} = 6t$

$\frac{1}{6t}\,dw = dt$

$= \int \frac{1}{6} e^w\,dw$

$= \frac{1}{6}\int e^w\,dw = \frac{1}{6}e^w + C$

$= \boxed{\frac{1}{6}e^{3t^2} + C}$

4. **[2 points]** Use the Fundamental Theorem of Calculus to evaluate the definite integral $\int_1^2 \frac{1}{\sqrt{t+2}}dt$ exactly.

$$\boxed{\begin{aligned} w &= t+2 \\ \frac{dw}{dt} &= 1 \\ dw &= dt \end{aligned}}$$

$\int \frac{1}{\sqrt{t+2}}\,dt = \int \frac{1}{\sqrt{w}}\,dw = \int w^{-\frac{1}{2}}\,dw$

$= 2w^{\frac{1}{2}} + C$

$= 2(t+2)^{1/2} + C$

$\int_1^2 \frac{1}{\sqrt{t+2}} = 2(t+2)^{1/2}\Big|_1^2 = 2(2+2)^{1/2} - 2(1+2)^{1/2}$

$= 2\cdot\sqrt{4} - 2\sqrt{3}$

$= \boxed{4 - 2\sqrt{3}}$