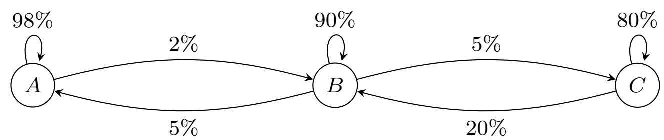
Name: ______________________________

Directions: Show all work. No credit for answers without work.

1. Three cities, A, B, and C share a population. Each year, a certain percentage of people living in a city stay or move to another city according to the diagram below. For example, in a year, 90% of the population of city B stays in city B, 5% moves to city A, and 5% moves to city C.



(a) **[10 points]** Given the initial population vector $\mathbf{x_0} = \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix}$, find a matrix A such that the populations after one year are given by $\mathbf{x_1} = A\mathbf{x_0}$.

(b) **[15 points]** Given that 15 million people in total live in cities A, B, and C, ind equilibrium populations (if they exist).

2. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transform that maps $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ to $\begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ to $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$.

   (a) [**10 points**] Determine the image of $\begin{bmatrix} -1 \\ 10 \end{bmatrix}$ under T.

   (b) [**5 points**] How many rows and columns does the standard matrix for T have?

3. [**10 points**] Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Express A as the product of elementary matrices.

4. **[10 points]** Suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ is a linearly dependent set, but removing any single vector in the set gives a linearly independent set of size $p-1$. Let $c_1, \ldots, c_p$ be scalars such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_p\mathbf{v}_p = \mathbf{0}$. What can you say about $c_1, \ldots, c_p$? Explain.

5. **[15 points]** Find the inverse of the following matrix.

$$\begin{bmatrix} 1 & 2 & -1 \\ -6 & -14 & -7 \\ 2 & 5 & 4 \end{bmatrix}$$

6. **[5 points]** Give geometric descriptions of all subspaces of $\mathbb{R}^2$.

7. **[2 parts, 10 points each]** Let A and B be the given matrices below. We are given that A and B are row-equivalent.

$$A = \begin{bmatrix} 16 & -3 & -47 & 60 & 10 \\ 1 & 0 & -2 & 3 & 0 \\ 5 & 4 & 10 & -1 & 3 \\ -5 & 1 & 15 & -19 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & -2 & 3 & 0 \\ 0 & 1 & 5 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for Col(A).

(b) Find a basis for Nul(A).