Name: ______________________________

Directions: Show all work. No credit for answers without work.

1. **[15 points]** Solve the following system of equations.

$$\begin{array}{rcrcrcr} 3x_1 & - & x_2 & + & 2x_3 & = & 12 \\ x_1 & + & x_2 & - & x_3 & = & -2 \\ 4x_1 & - & 5x_2 & + & 2x_3 & = & 1 \end{array}$$

2. **[5 parts, 2 points each]** True/False. If the statement is true, then explain why. If the statement is false, then edit the statement slightly to make it true.

   (a) In a matrix with 10 rows, replacing row 8 with $2(R7) - R8$ (i.e. 2 times row 7 minus row 8) is an elementary row operation.

   (b) Let A be a matrix and let A' be a matrix obtained from A by swapping two columns. The system $A\mathbf{x} = \mathbf{b}$ is consistent if and only if $A'\mathbf{x} = \mathbf{b}$ is consistent.

   (c) Every inconsistent system is nonhomogeneous.

   (d) The solution set for $A\mathbf{x} = \mathbf{b}$ is a translation of the solution set for $A\mathbf{x} = \mathbf{0}$.

   (e) Every matrix is row equivalent to a unique matrix in reduced echelon form.

3. **[10 points]** Determine the value(s) of h, if any, make the linear system represented by the following augmented matrix consistent.

$$\begin{bmatrix} 6 & -1 & 1 & 7 \\ 2 & h & -2 & -10 \\ 1 & 2 & 3 & 7 \\ 3 & 0 & 1 & 5 \end{bmatrix}$$

4. **[15 points]** Give an equation for the components of $\mathbf{b}$ that determines when the system $A\mathbf{x} = \mathbf{b}$ is consistent.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 8 & -4 & 12 \\ 1 & -2 & 3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

5. **[10 points]** Suppose that $\mathbf{v_1}, \ldots, \mathbf{v_n}$ are vectors in $\mathbb{R}^m$ that span $\mathbb{R}^m$. Give a careful argument that $n \geq m$.

6. **[10 points]** Besides $\mathbf{0}$, is there a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ that has no negative entries? Justify your answer.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} \qquad \mathbf{a}_2 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \qquad \mathbf{a}_3 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

7. **[15 points]** Express the solution set to $A\mathbf{x} = \mathbf{0}$ in parametric form.

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & -3 & -2 & 0 \\ 0 & 0 & 1 & 0 & 4 & 3 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

8. **[15 points]** Express the solution set to $A\mathbf{x} = \mathbf{b}$ in parametric form.

$$A = \begin{bmatrix} 5 & 20 & -5 & -2 & 1 \\ 2 & 8 & -2 & 1 & -1 \\ -3 & -12 & 3 & 0 & -1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 12 \\ 8 \\ -8 \end{bmatrix}$$