

Name: _____

Directions: Show all work. No credit for answers without work.

1. [2 points] Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$, where $\mathbf{b}_1 = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$. Given $\mathbf{x} = \begin{bmatrix} -17 \\ 2 \\ 11 \end{bmatrix}$, find $[\mathbf{x}]_{\mathcal{B}}$ if possible.

2. [1 point] What is the rank of a 4×5 matrix whose null space has dimension 3?

3. [1 point] Let A be an $n \times n$ matrix with two equal rows. What, if anything, can we conclude about $\det(A)$? Explain.

4. Compute the determinant of the following matrices.

(a) [1 point] $\begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix}$

(b) [1 point] $\begin{bmatrix} 1 & -1 & 4 \\ 1 & 3 & -2 \\ 4 & 7 & -1 \end{bmatrix}$

(c) [2 points] $\begin{bmatrix} 9 & 0 & 1 & 4 \\ 2 & -1 & 5 & 3 \\ 0 & 0 & 0 & 2 \\ 1 & 0 & 3 & -2 \end{bmatrix}$

(d) [2 points] $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 5 & 3 & 2 & 2 \\ -1 & 1 & 3 & 2 \\ -2 & 5 & 2 & 1 \end{bmatrix}$