Math343 **Quiz 8** **Oct 14, 2022**

Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[2 parts, 3 points each]** Find the inverses of the following matrices, if they exist. Except for part (a), use the row-reduction algorithm.

(a) $\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\det(A) = 2(-1) - (3)(-5) = -2 + 15 = 13$, so $A^{-1} = \boxed{\frac{1}{13}\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}}$

(b) $\begin{array}{c} \begin{matrix} 5 & 4 & 2 \end{matrix} \\ \begin{bmatrix} -2 & 4 & -3 \\ -8 & 17 & -14 \\ 3 & -6 & 5 \end{bmatrix} \end{array}$

$$\begin{bmatrix} -2 & 4 & -3 & 1 & 0 & 0 \\ -8 & 17 & -14 & 0 & 1 & 0 \\ 3 & -6 & 5 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 2 & 1 & 0 & 1 \\ -8 & 17 & -14 & 0 & 1 & 0 \\ 3 & -6 & 5 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 8 & 1 & 8 \\ 0 & 0 & -1 & -3 & 0 & -2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -2 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 8 & 1 & 8 \\ 0 & 0 & 1 & 3 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 0 & -5 & 0 & -3 \\ 0 & 1 & 0 & 2 & 1 & 4 \\ 0 & 0 & 1 & 3 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 & 2 & 5 \\ 0 & 1 & 0 & 2 & 1 & 4 \\ 0 & 0 & 1 & 3 & 0 & 2 \end{bmatrix}$$

So $\boxed{A^{-1} = \begin{bmatrix} -1 & 2 & 5 \\ 2 & 1 & 4 \\ 3 & 0 & 2 \end{bmatrix}}$

2. **[2 points]** Prove that if A is row-equivalent to an invertible matrix B, then A is also invertible.

Suppose A is row equivalent to B. Then there exist elementary matrices $E_1, \ldots, E_k$ such that

$$E_k \cdots E_1 A = B.$$

Since each E_i is invertible, it follows that $A = E_1^{-1} E_2^{-1} \cdots E_k^{-1} B$, and so A is the product of invertible matrices. Therefore A is invertible, and

$$A^{-1} = (E_1^{-1} E_2^{-1} \cdots E_k^{-1} B)^{-1} = B^{-1} E_k E_{k-1} \cdots E_1.$$

3. **[2 parts, 1 point each]** Elementary matrices.

(a) Give the elementary matrix E that, in a system with 4 equations, corresponds to the elementary row operation $R3 \leftarrow R3 + (2)(R2)$.

$$\boxed{E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

Rows 1, 2, & 4 stay the same

(b) Find the matrix E^{-1}.

Inverse operation is $R3 \leftarrow R3 + (-2)(R2)$, so

$$\boxed{E^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$