Name: ____________________________________

Directions: Show all work. No credit for answers without work.

1. **[2 parts, 2 points each]** Decide whether the given transformation is linear. Justify your answer.

   (a) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} x_1 + 1 \\ x_2 - 1 \end{bmatrix}$

   (b) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} \ln(2)x_1 \\ -x_2 \end{bmatrix}$.

2. **[1 point]** Let $T\colon \mathbb{R}^n \to \mathbb{R}^m$ be a linear transform, and let $\mathbf{v}_1, \dots \mathbf{v}_p$ be vectors in $\mathbb{R}^n$. Show that if $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ is a linearly dependent set, then $\{T(\mathbf{v}_1), \dots, T(\mathbf{v}_p)\}$ is linearly dependent.

3. **[2 parts, 2 points each]** Suppose that $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transform, let $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and let $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. We know that T maps $\mathbf{u}$ to $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and T maps $\mathbf{v}$ to $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$.

   (a) Find the image of $2\mathbf{u} - \mathbf{v}$ under T.

   (b) If possible, then find $T(\mathbf{w})$, where $\mathbf{w} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$. If not possible, then explain why not.

4. **[1 point]** Give the standard matrix for the transform $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ that rotates the plane by 45 degrees counter-clockwise.