Name: _____

Directions: Show all work. No credit for answers without work.

1. **[4 parts, 1 point each]** True/False. Justify your answers.

   (a) Let A be an $(m \times n)$ matrix and let $\mathbf{x}$ be a vector. The product $A\mathbf{x}$ is defined only if $\mathbf{x}$ has size n.

   (b) If $\mathbf{b}$ is a linear combination of $\mathbf{a}_1, \ldots, \mathbf{a}_p$, then so is $-\mathbf{b}$.

   (c) For all vectors $\mathbf{u}$ and $\mathbf{v}$, the set $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ is bigger than the set $\text{Span}\{\mathbf{u}\}$.

   (d) In most cases, when we choose two vectors $\mathbf{u}$ and $\mathbf{v}$ from $\mathbb{R}^3$, the sets $\text{Span}\{\mathbf{u}\}$ and $\text{Span}\{\mathbf{v}\}$ do not intersect.

2. **[2 points]** For $A = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 8 & 2 \\ 1 & 0 & -5 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 18 \\ 24 \\ 11 \end{bmatrix}$, solve $A\mathbf{x} = \mathbf{b}$.

3. **[2 parts, 2 points each]** Decide whether the vector $\mathbf{b}$ is a linear combination of the vectors $\mathbf{a}_1, \ldots \mathbf{a}_p$ given below.

(a) $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 4 \\ 6 \\ 9 \end{bmatrix}$

(b) $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 7 \\ 3 \\ -2 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} -8 \\ -2 \\ 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$