Math343 **Quiz 2** **Sep 2, 2022**

Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[3 points]** Find the general solution of the system with the following augmented matrix.

$$\begin{bmatrix} 2 & 4 & -1 & 1 & 13 \\ -1 & -2 & 3 & -1 & -14 \\ 3 & 6 & 1 & 0 & 7 \end{bmatrix}$$

$7 + (-3)(14) = 7 + (-6)(7)$
$= 7(-5)$

$$\begin{bmatrix} 2 & 4 & -1 & 1 & 13 \\ -1 & -2 & 3 & -1 & -14 \\ 3 & 6 & 1 & 0 & 7 \end{bmatrix} \xrightarrow[R1 \leftrightarrow R2]{R2 \cdot (-1)} \begin{bmatrix} 1 & 2 & -3 & 1 & 14 \\ 2 & 4 & -1 & 1 & 13 \\ 3 & 6 & 1 & 0 & 7 \end{bmatrix} \xrightarrow[R3 \pm (-3)R1]{R2 \pm (-2)R1} \begin{bmatrix} 1 & 2 & -3 & 1 & 14 \\ 0 & 0 & 5 & -1 & -15 \\ 0 & 0 & 10 & -3 & -35 \end{bmatrix}$$

$$\xrightarrow{R3 \pm (-2)R2} \begin{bmatrix} 1 & 2 & -3 & 1 & 14 \\ 0 & 0 & 5 & -1 & -15 \\ 0 & 0 & 0 & -1 & -5 \end{bmatrix} \xrightarrow{R3 \cdot (-1)} \begin{bmatrix} 1 & 2 & -3 & 1 & 14 \\ 0 & 0 & 5 & -1 & -15 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix} \xrightarrow[R2 \pm R3]{R1 \pm (-1)R3} \begin{bmatrix} 1 & 2 & -3 & 0 & 9 \\ 0 & 0 & 5 & 0 & -10 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\xrightarrow{R2 \cdot \frac{1}{5}} \begin{bmatrix} 1 & 2 & -3 & 0 & 9 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix} \xrightarrow{R1 \pm (3)R2} \begin{bmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\boxed{\begin{aligned} x_4 &= 5 \\ x_3 &= -2 \\ x_2 &\text{ free} \\ x_1 &= -2x_2 + 3 \end{aligned}}$$

2. **[2 points]** Are the two matrices given below row equivalent? Explain why or why not.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$$

Convert both A and B to reduced echelon form and compare.

$$\underline{A:} \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \qquad \underline{B:} \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$$

Since A and B have different reduced echelon forms, A and B are $\boxed{\text{not row equivalent.}}$

3. **[3 points]** Find a quadratic polynomial $f(t) = a + bt + ct^2$ such that $f(-1) = 9$, $f(1) = 5$, and $f'(-1) = -4$.

$f(-1)=9$: $\quad a + b(-1) + c(-1)^2 = 9 \quad \Rightarrow \quad a - b + c = 9$

$f(1)=5$: $\quad a + b(1) + c(1)^2 = 5 \quad \Rightarrow \quad a + b + c = 5$

$f'(-1)=-4$: $\quad b + 2c(-1) = -4 \quad \Rightarrow \quad b - 2c = -4$

$$\begin{bmatrix} 1 & -1 & 1 & 9 \\ 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -4 \end{bmatrix} \xrightarrow{R2 \pm (-1)R1} \begin{bmatrix} 1 & -1 & 1 & 9 \\ 0 & 2 & 0 & -4 \\ 0 & 1 & -2 & -4 \end{bmatrix} \xrightarrow{R2 \cdot \frac{1}{2}} \begin{bmatrix} 1 & -1 & 1 & 9 \\ 0 & 1 & 0 & -2 \\ 0 & 1 & -2 & -4 \end{bmatrix} \xrightarrow{R3 \pm (-1)R2} \begin{bmatrix} 1 & -1 & 1 & 9 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -2 & -2 \end{bmatrix}$$

$$\xrightarrow{R3 \cdot (-\frac{1}{2})} \begin{bmatrix} 1 & -1 & 1 & 9 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R1 \pm (-1)R3} \begin{bmatrix} 1 & -1 & 0 & 8 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R1 \pm R2} \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

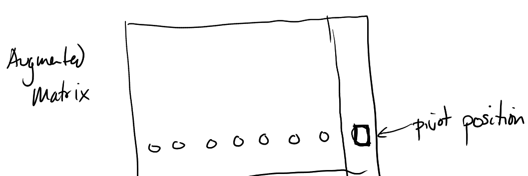
$a = 6$
$b = -2$
$c = 1$

$$f(t) = 6 - 2t + t^2$$

4. **[2 parts, 1 point each]** Pivot columns and number of solutions.

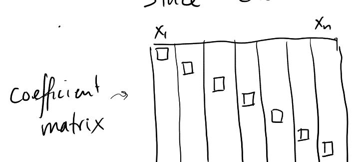
(a) Suppose that every column in the *augmented* matrix of a linear system contains a pivot position. What can you conclude about the number of solutions to the system? Explain.

$\boxed{\text{The system has no solutions}}$. Since the last column has a pivot position, the row containing this pivot position represents an equation whose LHS is zero and whose RHS is nonzero.



(b) Suppose that every column in the *coefficient* matrix of a linear system contains a pivot position. What can you conclude about the number of solutions to the system? Explain.

Since each column has a pivot position, it must be that the ith column has its pivot position in the i^{th} row. Let n be the number of columns in the coefficient matrix. If the matrix has n rows, then the system has one soln, as the LHS of row i is just x_i.



If the number of rows is more than n, then the rows below the n^{th} row may represent contradictory eqns ($0 =$ nonzero number) or trivial eqns ($0 = 0$), depending on the RHS.

$\boxed{\text{So the number of solutions is either } 0 \text{ or } 1.}$