Name: ______________________________

Directions: Show all work. No credit for answers without work.

1. **[2 points]** Given $\mathbf{y}$ and $\mathbf{v}$ below, decompose $\mathbf{y}$ as $\mathbf{y} = c\mathbf{v} + \mathbf{z}$ where c is a scalar and $\mathbf{z} \cdot \mathbf{v} = \mathbf{0}$.

$$\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$$

2. **[2 parts, 2 points each]** Define $\mathbf{v}_1, \mathbf{v}_2, \mathbf{y}$ as follows and let $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$.

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \\ 3 \end{bmatrix} \qquad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

(a) Find $\text{proj}_W(\mathbf{y})$.

(b) Find the distance from $\mathbf{y}$ to W.

3. **[1 point]** Let W be a subspace of $\mathbb{R}^n$. Suppose that $\mathbf{y} = \hat{\mathbf{y}}_1 + \mathbf{z}_1 = \hat{\mathbf{y}}_2 + \mathbf{z}_2$ where $\hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2 \in W$ and $\mathbf{z}_1, \mathbf{z}_2 \in W^{\perp}$. Prove that $\hat{\mathbf{y}}_1 = \hat{\mathbf{y}}_2$ and $\mathbf{z}_1 = \mathbf{z}_2$. [Hint: begin with $\hat{\mathbf{y}}_1 - \hat{\mathbf{y}}_2 = \mathbf{z}_2 - \mathbf{z}_1$ and take the dot product with an appropriate vector on both sides.]

4. **[3 parts, 1 point each]** True/False. In the following, A and B are $n \times n$ matrices. Justify your answer.

   (a) If A has orthogonal columns, then A^T also has orthogonal columns.

   (b) A has orthonormal columns if and only if $A^T A = I$.

   (c) If A and B have orthonormal columns, then so does AB.