Math343 **Quiz 1** **Aug 26 2022**

Name: Solutions

Directions: All answers must be handwritten in your own hand. Show all work. No credit for answers without work.

1. **[3.5 points]** Determine if the following system is consistent.

$$\begin{aligned} 2x_1 - x_2 - 4x_3 &= 0 \\ x_1 - x_2 - 3x_3 &= 2 \\ 3x_1 + 2x_2 + x_3 &= -11 \end{aligned}$$

2. $$\begin{bmatrix} 2 & -1 & -4 & 0 \\ 1 & -1 & -3 & 2 \\ 3 & 2 & 1 & -11 \end{bmatrix} \overset{R1 \leftrightarrow R2}{\rightsquigarrow} \begin{bmatrix} 1 & -1 & -3 & 2 \\ 2 & -1 & -4 & 0 \\ 3 & 2 & 1 & -11 \end{bmatrix}$$

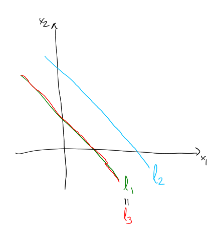
$$\begin{matrix} R2 \mathrel{+}= (-2)R1 \\ R3 \mathrel{+}= (-3)R1 \\ \rightsquigarrow \end{matrix} \begin{bmatrix} 1 & -1 & -3 & 2 \\ 0 & 1 & 2 & -4 \\ 0 & 5 & 10 & -17 \end{bmatrix} \overset{R3 \mathrel{+}= (-5)R2}{\rightsquigarrow} \begin{bmatrix} 1 & -1 & -3 & 2 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$\Longrightarrow \quad \begin{aligned} x_1 - x_2 - 3x_3 &= 2 \\ x_2 + 2x_3 &= -4 \\ 0 &= 3 \end{aligned}$$

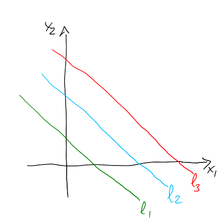
System is $\boxed{\text{inconsistent}}$.

2. **[3 points]** A linear equation of the form $ax_1 + bx_2 = c$ is *degenerate* if $a = 0$ and $b = 0$. Let E_1, E_2, and E_3 be nondegenerate linear equations in the variables x_1 and x_2. Suppose that E_1 and E_2 form an inconsistent system, and also E_2 and E_3 form an inconsistent system. What can you conclude about the size of the solution set of the system formed by E_1 and E_3? Justify your answer.

If E_1 and E_2 form an inconsistent system, then the corresponding lines ℓ_1 and ℓ_2 are parallel. Similarly, ℓ_2 and ℓ_3 are parallel. Therefore ℓ_1 and ℓ_3 are parallel. Hence either $\ell_1 = \ell_3$, and the system formed by E_1 and E_3 has infinitely many solutions, or ℓ_1 and ℓ_3 do not intersect and this system has no solutions.



Case 1: Infinitely many Solutions



Case 2: No solutions

3. **[3.5 points]** Solve the following system.

$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & - & x_3 & = & 10 \\ 2x_1 & + & x_2 & + & x_3 & = & -1 \\ -2x_1 & & & + & x_3 & = & -1 \end{array}$$

1. $\begin{bmatrix} 1 & 2 & -1 & 10 \\ 2 & 1 & 1 & -1 \\ -2 & 0 & 1 & -1 \end{bmatrix} \underset{\leadsto}{\overset{R2 \pm (-2)(R1)}{R3 \pm 2(R1)}} \begin{bmatrix} 1 & 2 & -1 & 10 \\ 0 & -3 & 3 & -21 \\ 0 & 4 & -1 & 19 \end{bmatrix}$

$\overset{R2 \cdot (-\frac{1}{3})}{\leadsto} \begin{bmatrix} 1 & 2 & -1 & 10 \\ 0 & 1 & -1 & 7 \\ 0 & 4 & -1 & 19 \end{bmatrix} \overset{R3 \pm (-4)(R2)}{\leadsto} \begin{bmatrix} 1 & 2 & -1 & 10 \\ 0 & 1 & -1 & 7 \\ 0 & 0 & 3 & -9 \end{bmatrix}$

$\overset{R3 \cdot \frac{1}{3}}{\leadsto} \begin{bmatrix} 1 & 2 & -1 & 10 \\ 0 & 1 & -1 & 7 \\ 0 & 0 & 1 & -3 \end{bmatrix} \underset{\leadsto}{\overset{R2 \pm R3}{R1 \pm R3}} \begin{bmatrix} 1 & 2 & 0 & 7 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{bmatrix}$

$\overset{R1 \pm (-2)R2}{\leadsto} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{bmatrix} \Longrightarrow$

$x_1 = -1$

$x_2 = 4$

$x_3 = -3$