Directions: You may work to solve these problems in groups, but all written work must be your own. Show all work; no credit for solutions without work..

1. True/False. In the following, A, B, and C are matrices for which the given expressions are defined. Justify your answer.

   (a) Each column of AB is a linear combination of the columns of B using weights from the corresponding column of A.

   (b) The second row of AB is the second row of A multiplied on the right by B.

   (c) $A^T + B^T = (A + B)^T$.

   (d) $(AB)C = (AC)B$

   (e) $(AB)^T = A^T B^T$

   (f) If A can be row reduced to the identity matrix, then A must be invertible.

   (g) Each elementary matrix is invertible.

   (h) If A is invertible, then the elementary row operations that reduce A to the identity I_n also reduce A^{-1} to I_n.

   (i) If the columns of an $n \times n$ matrix A are linearly independent, then they span $\mathbb{R}^n$.

   (j) A square matrix with two identical columns is singular.

2. [2.1.32] Let A be an $(m \times n)$ matrix and let D be an $(n \times m)$ matrix such that $AD = I_m$. Show that for each $\mathbf{b} \in \mathbb{R}^m$, the equation $A\mathbf{x} = \mathbf{b}$ has a solution. [Hint: Think about the equation $AD\mathbf{b} = \mathbf{b}$.] Also show that $m \leq n$.

3. [2.2.{39-42}] Find the inverses of the following matrices, if they exist. Use the row reduction algorithm.

   (a) $\begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}$

   (b) $\begin{bmatrix} 5 & 10 \\ 4 & 7 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$

4. [2.3.{1,2,6,7,8}] Using as few calculations as possible, determine if the following matrices are invertible. (Do not fully compute any inverses.) Justify your answers.

   (a) $\begin{bmatrix} 5 & 7 \\ -3 & -6 \end{bmatrix}$

   (b) $\begin{bmatrix} -4 & 6 \\ 6 & -9 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix}$

   (d) $\begin{bmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 8 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix}$

   (e) $\begin{bmatrix} 1 & 3 & 7 & 8 \\ 0 & 2 & 1 & 5 \\ 0 & 0 & 8 & 8 \\ 0 & 0 & 0 & 3 \end{bmatrix}$