Directions: You may work to solve these problems in groups, but all written work must be your own. Show all work; no credit for solutions without work..

1. [1.9] True/False. Justify your answer.
   (a) If A is a 3×2 matrix, then the transformation $x \mapsto Ax$ cannot be one-to-one.
   (b) If A is a 3×2 matrix, then the transformation $x \mapsto Ax$ cannot map $\mathbb{R}^2$ onto $\mathbb{R}^3$.

2. [1.10] Two nations, A and B, occupy an island. Each year, 10% of A's population moves to B and 25% of B's population moves to A. The rest stay put.
   (a) If A begins with 30 million people and B begins with 40 million people, what will their populations be after one, two, and three years?
   (b) Given that 70 million people live on the island, do there exist *stable* population levels for A and B that would stay the same year after year? Either find stable population levels or explain why they do not exist.

3. [2.1.1] Compute each matrix sum or product if it is defined. If undefined, then explain why.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}$$

   (a) $-2A$
   (b) $B - 2A$
   (c) AC
   (d) CD

4. [2.1.11] Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 5 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$. Compute AD and DA. Explain how the columns or rows of A change when A is multiplied by D on the right or on the left. Find a 3×3 matrix B, not the identity matrix or the zero matrix, such that $AB = BA$.

5. [2.1.9] Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What value(s) of k, if any, will make $AB = BA$?

6. [2.1.12] Let $A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$. Construct a 2×2 matrix B such that AB is the zero matrix. Use two different nonzero columns for B.