Directions: You may work to solve these problems in groups, but all written work must be your own. Show all work; no credit for solutions without work..

1. [1.6.6] Balance the following chemical reaction.

$$Na_3PO_4 + Ba(NO_3)_2 \to Ba_3(PO_4)_2 + NaNO_3$$

2. [1.7.{1,3,4}] Determine if the vectors are linearly independent. Justify your answer.

   (a) $\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 6 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \end{bmatrix}$

   (c) $\begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 8 \end{bmatrix}$

3. [1.7.{9,10}] For which values of h is $\mathbf{v}_3$ in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$? For which values of h is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly dependent?

   (a) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -3 \\ 10 \\ -6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2 \\ -7 \\ h \end{bmatrix}$

   (b) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2 \\ -10 \\ h \end{bmatrix}$

4. True/False. Justify your answers.

   (a) Two vectors are linearly dependent if and only if they lie on a line through the origin.

   (b) If S is a linearly dependent set, then each vector in S is a linear combination of the other vectors in S.

   (c) If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent.

   (d) If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent but $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, then $\mathbf{z} \in$ Span$\{\mathbf{x}, \mathbf{y}\}$.

5. Let $\mathbf{u_1}, \ldots, \mathbf{u_n}$ be vectors, and suppose that $\mathbf{v}$ is a vector that can be obtained as a linear combination of $\mathbf{u_1}, \ldots, \mathbf{u_n}$ in two different ways. Prove that $\mathbf{u_1}, \ldots, \mathbf{u_n}$ are linearly dependent.