Directions: You may work to solve these problems in groups, but all written work must be your own. Show all work; no credit for solutions without work..

1. [1.4.14] Let $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$. Is $\mathbf{u}$ in the subset of $\mathbb{R}^3$ spanned by the columns of A? Why or why not?

2. [1.4.{15,16}] For the matrices A and vectors $\mathbf{b}$ below, show that the equation $A\mathbf{x} = \mathbf{b}$ does not have a solution for all possible $\mathbf{b}$, and describe the set of all $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ does have a solution.

   (a) $A = \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$.

   (b) $A = \begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

3. [1.4.42] Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$ when n is less than m?

4. [1.5.{7,11}] Given a matrix A and a vector $\mathbf{b}$, describe all solutions to $A\mathbf{x} = \mathbf{b}$ in parametric form.

   (a) $A = \begin{bmatrix} 1 & 3 & -3 & 7 \\ 0 & 1 & -4 & 5 \end{bmatrix}$, $\mathbf{b} = \mathbf{0}$

   (b) $A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, $\mathbf{b} = \mathbf{0}$

   (c) $A = \begin{bmatrix} 1 & 0 & 3 & 1 & -1 \\ 2 & -1 & 2 & 0 & 1 \\ -1 & 1 & 3 & 3 & 2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$

5. Let A be an $(m \times n)$ matrix, let $\mathbf{v} \in \mathbb{R}^n$, and let c be a scalar. Prove that $A(c\mathbf{v}) = c(A\mathbf{v})$.