Directions: You may work to solve these problems in groups, but all written work must be your own. Show all work; no credit for solutions without work.

1. [5.1.{9-16}] Find a basis for the eigenspace corresponding to each listed eigenvalue.

   (a) $\begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$, $\lambda = 1, 5$

   (b) $\begin{bmatrix} 4 & -2 \\ -3 & 9 \end{bmatrix}$, $\lambda = 10$

   (c) $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$, $\lambda = 1, 2, 3$

   (d) $\begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$, $\lambda = 3$

2. True/False. Justify your answers. In the following, A is an $(n \times n)$-matrix.

   (a) If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector $\mathbf{x}$, then λ is an eigenvalue of A.

   (b) The matrix A is invertible if and only if 0 is an eigenvalue of A.

   (c) To find the eigenvalues of A, reduce A to echelon form.

   (d) If $\mathbf{v}$ is an eigenvector with eigenvalue 2, then $2\mathbf{v}$ is an eigenvector with eigenvalue 4.

   (e) An eigenspace of A is a null space of a certain matrix.

3. [5.1.33] Let λ be an eigenvalue of an invertible matrix A. Show that λ^{-1} is an eigenvalue of A^{-1}.

4. [5.2.{1-14}] Find the characteristic polynomial and eigenvalues of the matrices below.

   (a) $\begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 4 & -3 \\ -4 & 2 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & -1 \\ 0 & 6 & 0 \end{bmatrix}$

   (d) $\begin{bmatrix} 6 & -2 & 0 \\ -2 & 9 & 0 \\ 5 & 8 & 3 \end{bmatrix}$

5. [5.2.18] Find h in the matrix A below such that the eigenspace for $\lambda = 5$ is two-dimensional.

$$\begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. [5.2.20] Use a property of determinants to show that A and A^T have the same characteristic polynomial.