Name: _____

**Directions:** Show all work. Answers without work generally do not earn points. Unless stated otherwise, answers may be left in terms of factorials and binomial coefficients. Your answers should not involve sums with more than a few terms.

1. [**3 parts, 6 points each**] Let $\Sigma = \{a, b\}$. Construct DFAs for the following languages.

   (a) $\{w \in \Sigma^* : w \text{ has length at least } 2\}$
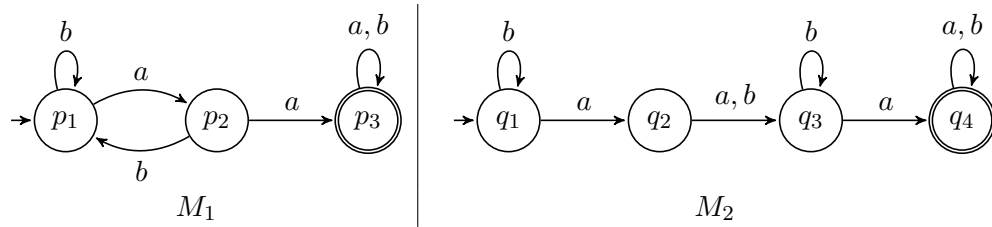
   (b) $\{w \in \Sigma^* : w = a^n b x \text{ for some odd integer } n \text{ and some string } x\}$

   (c) $\{w \in \Sigma^* : w \text{ does not contain the same symbol three times consecutively}\}$

2. [**1 point**] Fill in the blanks: In the formal definition of a DFA, the transition function $\delta$ has type
$$\delta : \underline{\hspace{3cm}} \rightarrow \underline{\hspace{3cm}}.$$

3. Let $\Sigma = \{a, b\}$. The DFAs $M_1$ and $M_2$ are pictured below.



(a) [**6 points**] Give simple, English descriptions of the languages computed by $M_1$ and $M_2$.

(b) [**10 points**] Construct a DFA that accepts a string $w$ if and only if both $M_1$ and $M_2$ accept $w$. (Hint: since many states would be unreachable in the product construction, it is more efficient to find successor states dynamically, only for those states that are reachable.)

4. [**6 parts, 3 points each**] True/False. For each statement below, decide if the statement is true or false, and justify your answer.

   (a) The language $A$ is regular if and only if $A$ is computed by some DFA.

   (b) The language $A$ is regular if and only if $A$ is computed by some NFA.

   (c) If $A$ is the language computed by an NFA $N$, then we obtain an NFA for the complement language $\overline{A}$ by inverting the accepting and rejecting states in $N$.

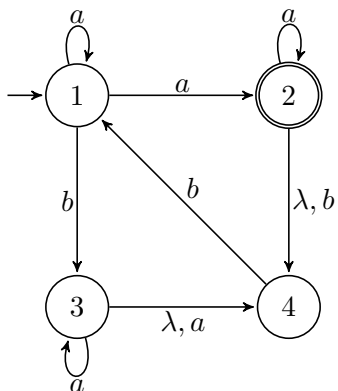   (d) Let $\Sigma = \{a, b\}$. There is a DFA that computes the language $\{a^n b^n : n \geq 0\}$.

   (e) If $A$ and $B$ are regular languages, then $A - B$ is also a regular language.

   (f) Let $\Sigma = \{a, b\}$. There is a DFA that computes the language

$$\{w \in \Sigma^* : w = xy \text{ for some strings } x \text{ and } y \text{ with } |x| = |y|\}.$$

   (Recall that if $x$ is a string in $\Sigma^*$, then $|x|$ denotes the length of $x$.)

5. Let $N$ be the NFA pictured below.



(a) [**6 points**] Complete the transition table below.

|   | $\lambda^*$ | $\lambda^* a$ | $\lambda^* a \lambda^*$ | $\lambda^* b$ | $\lambda^* b \lambda^*$ |
|---|---|---|---|---|---|
| 1 |   |   |   |   |   |
| 2 |   |   |   |   |   |
| 3 |   |   |   |   |   |
| 4 |   |   |   |   |   |

(b) [**10 points**] Convert $N$ to a DFA.

6. [**2 parts, 5 points each**] Let $\Sigma = \{0, 1, 2\}$. Construct NFAs for the following languages with at most the prescribed number of states.

   (a) A 5-state NFA for $\{w \in \Sigma^* \colon |w| \geq 2 \text{ and starts and ends with the same symbol}\}$

   (b) A 4-state NFA for $\{w \in \Sigma^* \colon w \text{ does not contain all 3 input symbols}\}$

7. [**5 points**] Let $\Sigma = \{a, b, c\}$ and let
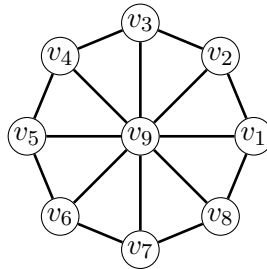
$$A = \{w \in \Sigma^* \colon \#a(w) \text{ is even}\}$$
$$B = \{w \in \Sigma^* \colon \#b(w) \text{ is even}\}$$
$$C = \{w \in \Sigma^* \colon \#c(w) \text{ is even}\}.$$

Construct an NFA for the language $ABC$, where $ABC = \{xyz \in \Sigma^* \colon x \in A, y \in B, \text{ and } z \in C\}$. (Do not convert to a DFA.)

8. [**4 points**] How many edges are there in $K_{50}$, the complete graph on 50 vertices?

9. [**3 parts, 4 points each**] Let $G$ be the following graph with 9 vertices.



(a) What are the degrees of the vertices in $G$?

(b) How many times does $C_3$ appear as a subgraph of $G$?

(c) In total, how many cycles does $G$ contain?

Scratch Paper