Name: _____

Directions: Show all work. Answers without work generally do not earn points. Unless stated otherwise, answers may be left in terms of factorials and binomial coefficients. Your answers should not involve sums with more than a few terms.

1. **[10 parts, 2 points each]** Let $A = \{\varnothing, \{1, 2\}, \{1\}, \{2\}\}$, $B = \{2, 1\}$, and $C = \{1, \{1\}, \{2\}\}$. For True/False questions, **write the whole word**. No work/explanation necessary.

   (a) Determine $|A|$, $|B|$, and $|C|$.

   (b) Determine $A \cap B$.

   (c) Determine $B \cup C$.

   (d) Determine $C - B$.

   (e) Determine $A \triangle C$.

   (f) True or False: $B \in A$.

   (g) True or False: $B \subseteq A$.

   (h) True or False: $B \in \mathcal{P}(C)$.

   (i) True or False: $A \subseteq \mathcal{P}(B)$.

   (j) True or False: $B \cap C = 1$.

2. **[4 parts, 2 points each]** Let $A = \{(1, 1)\}$ and $B = \{(1, 2), (3, 3)\}$.

   (a) Determine $|A|$ and $|B|$.

   (b) Determine $A \times B$.

   (c) Determine $\mathcal{P}(A)$.

   (d) Determine B^0.

3. **[3 parts, 3 points each]** Let $C = \{1, 2, \ldots, 10\}$. In terms of C, give the set that best models the space of all possibilities for each situation described below. For example, if the situation is "Joe thinks of a number between 1 and 10", then the appropriate set is just C.

   (a) Joe and Alice both think of numbers between 1 and 10.

   (b) A computer initializes a game of minesweeper on a (10×10) board, where each of the 100 cells might or might not contain a bomb.

   (c) A pizza restaurant offers 10 different toppings, numbered from 1 through 10 on the menu. Two people order pizzas from the restaurant, each with a particular set of toppings.

4. Are the following sets countable or not? If countable, then describe how to enumerate the set. If uncountable, then justify your answer by adapting Cantor's Diagonalization Argument.

   (a) **[5 points]** The set of integers $\mathbb{Z}$.

   (b) **[5 points]** The set $\mathbb{Z} \times \mathbb{Z}$.

   (c) **[6 points]** The set $\{A\colon\ A \text{ is a finite set of integers}\}$.

5. A 4-digit ATM pin is selected at random. Let A be the event that the first digit is even, and let B be the event that at least one of the digits is 7.

   (a) **[2 points]** What is the sample space Ω? What is $|\Omega|$?

   (b) **[6 points]** Determine $\Pr(A)$ and $\Pr(B)$.

   (c) **[6 points]** Determine $\Pr(A|B)$ and $\Pr(B|A)$.

   (d) **[2 points]** Are the events A and B pos. correlated, neg. correlated, or independent?

6. Recall that a standard deck of cards has one card for each rank/suit pair, where the ranks are [ace, 2 through 10, jack, queen, king], and the suits are [clubs, hearts, diamonds, spades]. A 5-card poker hand is dealt from a freshly shuffled deck. Let A be the event that the hand contains exactly one ace and let B be the event that the hand contains exactly one spade.

   (a) **[5 points]** Determine $\Pr(A)$ and $\Pr(B)$.

   (b) **[5 points]** Determine $\Pr(A|B)$.

7. **[6 points]** Suppose that 90% of the population exceeds the speed limit regularly when driving. A sociology study finds that, when asked, a non-speeder will always deny speeding. However, 20% of the time, a speeder will falsely claim not to speed. John claims that he is not a speeder. What is the probability that John is not a speeder?

8. **[3 parts, 3 points each]** Let $\Sigma = \{0, 1\}$. We define the following languages:

$$A = \{w \in \Sigma^* : \ w \text{ has even length}\}$$
$$B = \{w \in \Sigma^* : \ w \text{ has an odd number of ones}\}$$

   (a) Give an example of a string in $A \cap B$.

   (b) Describe the relationship between A and AA. Justify your answer.

   (c) Is there a string in $B \cap BB$? Give an example or explain why not.

9. Let $\Sigma = \{0, 1\}$. We define a language A over Σ recursively as follows.

   (1) $\lambda \in A$
   (2) If $x \in A$, then $0x \in A$.
   (3) If $x \in A$, then $1x1 \in A$.

   (a) **[3 points]** Is $0110 \in A$? Explain why or why not.

   (b) **[3 points]** Is $1001 \in A$? Explain why or why not.

   (c) **[5 bonus points (tricky, finish others first)]** Give a simple description of A.

Scratch Paper