Name: Solutions

Directions: Show all work. No credit for answers without work. Unless otherwise specified, you may leave your answers in terms of factorials and binomial/multinomial coefficients.

1. How many ways are there to rearrange the letters in the word 'MINIMUM':

   (a) [**2 points**] with no additional restrictions?

7 letters:

M: 3
I: 2
N: 1
U: 1

$$\boxed{\frac{7!}{(3!)(2!)(1!)(1!)}} = \boxed{420}$$

   (b) [**2 points**] so that no two M's are consecutive? For example, 'MINIMUM' counts but 'MINIUMM' and 'INIUMMM' does not.

Rule of Product:

1. Arrange INIU $\qquad n_1 = \frac{4!}{2!} = 12$

2. Insert 3 M's into 5 spaces $\qquad n_2 = \binom{5}{3} = \frac{5!}{2!\cdot 3!} = 10$

So total # is $12 \cdot 10$ or $\boxed{120}$.

   (c) [**1 point**] so that the 'U' is placed between the two 'I's, not necessarily consecutively? For example, 'MINUIMM' counts but 'MINIMUM' does not.

I M N U I M M $\longleftrightarrow$ Z M N Z Z M M

such arrangements = # arrangements M Z N Z M Z M

7 letters
M: 3
N: 1
Z: 3

$$= \boxed{\frac{7!}{(3!)(3!)(1!)}}$$

$$= \frac{7\cdot 6\cdot 5\cdot 4\cdot \cancel{3}\cdot \cancel{2}\cdot 1}{(\cancel{3!})\ (3!)} = \boxed{140}$$

2. [**1 point**] A carnival has a prize system where each token can be redeemed for a prize. There are 11 prizes available. Irene has won 3 prize tokens. Assuming she wants three different prizes, how many ways are there for her to redeem her tokens? **Express your answer as a simplified, concrete number.**

$$\binom{11}{3} = \frac{11!}{3!\,8!} = \frac{11\cdot 10\cdot 9\cdot \cancel{8!}}{3!\cdot \cancel{8!}} = \frac{11\cdot \overset{5}{\cancel{10}}\cdot \overset{3}{\cancel{9}}}{\cancel{3}\cdot\cancel{2}\cdot 1} = 11\cdot 15 = \boxed{165}$$

3. [**2 parts, 2 points each**] Poker hands. Recall that a standard deck has 52 cards: one for each suit/rank pair, where the 4 suits are spades, hearts, diamonds, clubs and the 13 ranks are ace, 2 through 10, jack, queen, and king. A poker hand is a set of 5 cards from the deck.

   (a) How many poker hands have all distinct ranks? For example, the hand $\{4S, 6S, 8S, 10H, QD\}$ counts but $\{4S, 6S, 6C, 10H, QD\}$ does not.

Rule of Product:

1. Choose 5 distinct ranks $\quad n_1 = \binom{13}{5}$

2. For each rank, choose a suit $\quad n_2 = \underset{\text{1st card}}{4}\cdot \cdots \cdot \underset{\text{5th card}}{4} = 4^5$

So total # is $\binom{13}{5}\cdot 4^5 = 1287\cdot 1024 = \boxed{1,317,888}$

   (b) A *face card* is a card whose rank is jack, queen, or king. How many poker hands have at least 1 face card? For example the hand $\{4S, 6S, 8S, 10H, QD\}$ counts but $\{4S, 6S, 6C, 10H, AD\}$ does not.

Count the complement. Note: the deck has $3\cdot 4$ or 12 face cards, and $52-12$ or 40 non-face cards.

total # hands: $\binom{52}{5}$

hands with no face card: $\binom{40}{5}$

hands with at least 1 face card: $\boxed{\binom{52}{5}-\binom{40}{5}} = \boxed{1,940,952}$