Name: _____

Directions: Show all work. No credit for answers without work. You may leave your answers in terms of factorials and, when necessary, sums with a few terms.

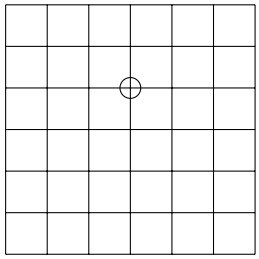
1. **[3 parts, 2 points each]** How many ways are there to arrange the letters of 'TRIATHLETE':

   (a) with no additional restrictions?

   (b) that start with at least two T's? (For example, both 'TTRIAHLETE' and 'TTTRIAHLEE' count.)

   (c) that have the 'A' between the two 'E's (not necessarily consecutively)? (For example, both 'TRIETHLATE' and 'TRIEAETHLT' count.)

2. **[2 points]** Recall that each step in a *lattice path* increases one of the coordinates by 1. Out of all lattice paths from $(0, 0)$ to $(6, 6)$, determine the fraction that pass through the point $(3, 4)$.



3. **[2 points]** Suppose that $n \geq 2$. How many ways are there to arrange the integers in $\{1, \ldots, n\}$ so that 1 and n are not next to each other? For example, if $n = 5$, then 2 3 5 4 1 counts but 2 3 5 1 4 does not.