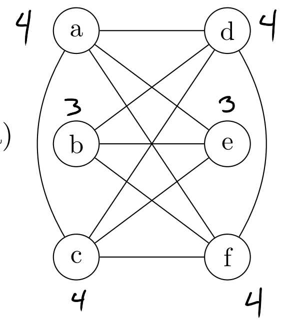
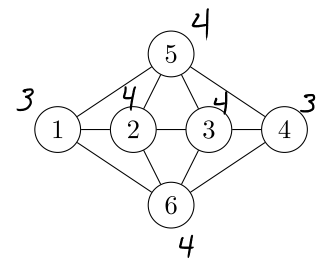
Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[2 parts, 2 points each]** Decide whether the following pairs of graphs are isomorphic. If they are isomorphic, give the function that establishes the isomorphism. If not, explain why.

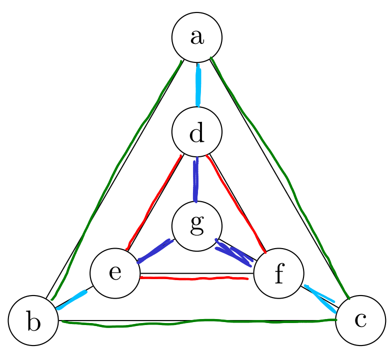
(a)

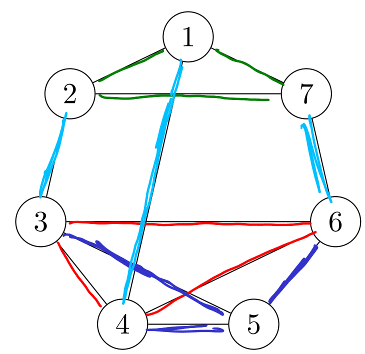




No, these graphs are $\boxed{\text{not isomorphic}}$. On the left, the two vertices of degree 3, b and e, are adjacent. On the right, the two vertices of degree 3, 1 and 4, are not adjacent.

(b)





These graphs are $\boxed{\text{isomorphic}}$. There are 6 isomorphisms. One of them is given by

Left	a	b	c	d	e	f	g
Right	1	2	7	4	3	6	5

The others are given by

$$g \longleftrightarrow 5$$

$$\{(a,d),\ (b,e),\ (c,f)\} \longleftrightarrow \{(1,4),\ (2,3),\ (7,6)\}$$

2. **[2 points]** Let G be a planar embedding of a connected graph with 95 vertices and 158 edges. Including the infinite region, how many regions are there?

By Euler's Formula, $n - a + r = 2$

$$95 - 158 + r = 2$$

$$r = 2 + 158 - 95 = 160 - 95 = 65.$$

So there are $\boxed{65}$ regions.

3. **[2 parts, 2 points each]** A graph G is *self-complementary* if G is isomorphic to its complement $\overline{G}$.

   (a) How many edges are there in an n-vertex self-complementary graph?

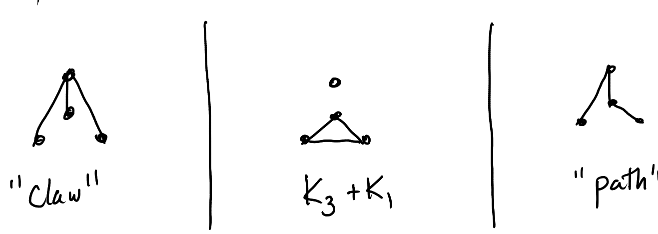
Since G has n vertices, we have that $|E(G)| + |E(\overline{G})| = \binom{n}{2}$.

Since $G \cong \overline{G}$, we have $|E(\overline{G})| = |E(G)|$.

Therefore $2|E(G)| = \binom{n}{2}$ and so $|E(G)| = \boxed{\frac{1}{2}\binom{n}{2}} = \boxed{\frac{n(n-1)}{4}}$.

   (b) Give an example of a 4-vertex self-complementary graph.

By part (a), such a graph has $\frac{4(3)}{4}$ or 3 edges. Up to isomorphism, there are 3 distinct 4-vertex 3-edge graphs:



Of these, only the path is self-complementary: