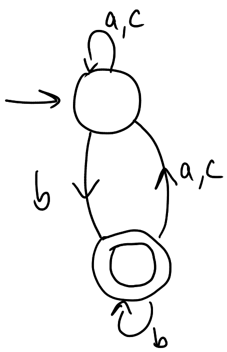
Name: Solutions

Directions: Show all work. No credit for answers without work.

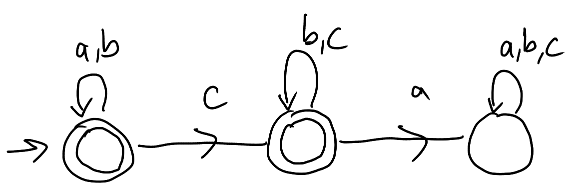
1. Let $\Sigma = \{a, b, c\}$. We define the following languages.

$$A = \{w \in \Sigma^* : w \text{ ends with a b}\}$$
$$B = \{w \in \Sigma^* : \text{every } a \text{ in } w \text{ appears before every } c\}$$

   (a) [**2 points**] Construct a DFA for A.

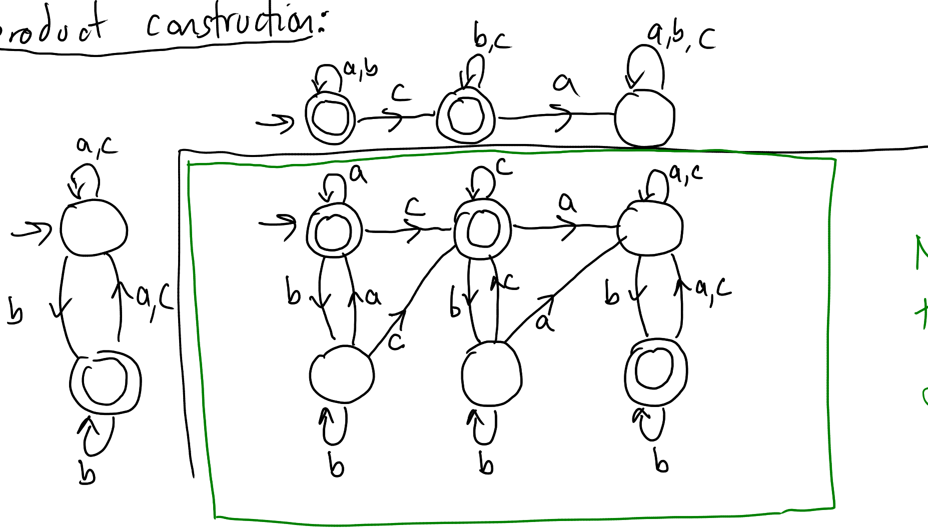


   (b) [**2 points**] Construct a DFA for B



   (c) [**3 points**] Construct a DFA for $A \triangle B$. (Recall that $A \triangle B = (A - B) \cup (B - A)$.)

Use product construction:



Note: this is the minimum DFA for $A \triangle B$.

2. **[3 points]** Let $\Sigma = \{0, 1\}$ and let $A = \{w \in \Sigma^* :$ the number of zeros and ones in w is not equal$\}$. Show that A is not a regular language. (Your argument should mostly use English sentences.)

Suppose for a contradiction that M is a DFA with $L(M) = A$, and let n be the number of states in M.

Consider the words $w_0, w_1, \ldots, w_n$, where $w_i = 0^i$. Since this is a list of $n+1$ words and M has just n states, there exist i and j such that $i < j$ but M on w_i and M on w_j end in the same state. It follows that M on $w_i 1^i$ and M on $w_j 1^i$ also end in the same state.

So M either accepts both $w_i 1^i$ and $w_j 1^i$, or M rejects both of these words. But $w_i 1^i = 0^i 1^i \notin A$ and $w_j 1^i = 0^j 1^i \in A$, and so $L(M) \neq A$, a contradiction. Therefore A is not regular.