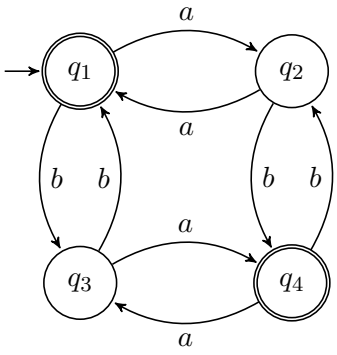
Name: Solutions

Directions: Show all work. No credit for answers without work.

1. Let $\Sigma = \{a, b\}$, and let M be the following automaton.



(a) **[1 point]** List the sequence of states of M on the string w, where $w = abbaa$. Is $w \in L(M)$?

w: $q_1 \xrightarrow{a} q_2 \xrightarrow{b} q_4 \xrightarrow{b} q_2 \xrightarrow{a} q_1 \xrightarrow{a} q_2$: $q_1, q_2, q_4, q_2, q_1, q_2$

Since q_2 is not an accepting state, $w \notin L(M)$.

(b) **[2 points]** Give a simple description for $L(M)$.

$L(M) = \{ w \in \Sigma^* :$ the parity of a's and b's in w is the same $\}$

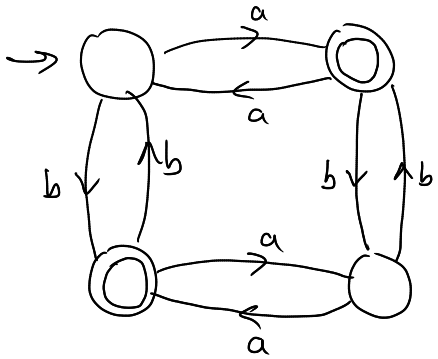
or

$L(M) = \{ w \in \Sigma^* :$ either w has an even number of a's and b's or w has an odd number of a's and b's $\}$

or

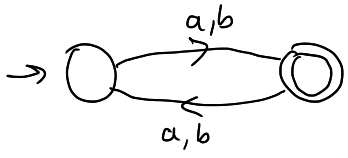
$L(M) = \{ w \in \Sigma^* :$ the length of w is even $\}$

(c) **[1 point]** Construct a machine M' with the property that $L(M') = \overline{L(M)}$.



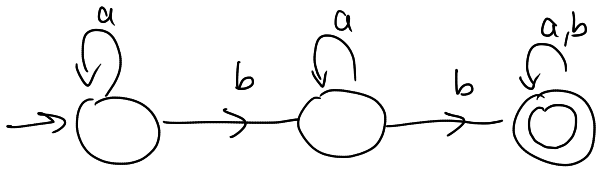
Interchange accepting and rejecting states.

or, using (b)

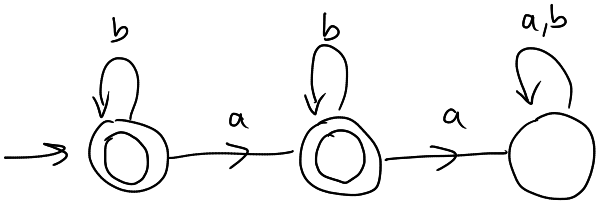


2. **[3 parts, 2 points each]** Let $\Sigma = \{a, b\}$. Construct (deterministic) finite automatons for the following languages over Σ.

   (a) $\{w \in \Sigma^* : w \text{ has at least two } b\text{'s}\}$



   (b) $\{w \in \Sigma^* : w \text{ has at most one } a\}$



   (c) $\{w \in \Sigma^* : w \text{ has at least two } b\text{'s and at most one } a\}$