Directions: You may work to solve these problems in groups, but all written work must be your own. See "Guidelines and advice" on the course webpage for more information.

1. Suppose that 15% of the population is infected with a virus. A test is developed which gives a positive result 98% of the time on an infected person and a positive result just 7% of the time on someone who is not infected. Mary tests positive for the virus. What is the probability Mary is actually infected?

2. Let $\Sigma = \{0, 1\}$, let $A = \bigcup_{k=0}^{2} \Sigma^k$, and let $B = \bigcup_{k=0}^{4} \Sigma^k$.

   (a) List the strings in A. What is $|A|$?

   (b) What is $|B|$?

   (c) Recall that $AB = \{xy \mid x \in A \text{ and } y \in B\}$. Describe the members of AB. What is $|AB|$?

3. Let $\Sigma = \{0, 1\}$. We define languages A, B, and C as follows:

$$A = \{w \in \Sigma^* : \ w \text{ contains more zeros than ones}\}$$
$$B = \{w \in \Sigma^* : \ w \text{ contains more ones than zeros}\}$$
$$C = \Sigma^*.$$

   (a) Give an example of a string x that belongs to the language A and a string y that *does not* belong to the language A.

   (b) Give a description of the language $A \cup B$.

   (c) Give a description of the language $A \cap B$.

   (d) Give a description of the language $\overline{A \cup B}$.

   (e) Give a description of the language AA.

   (f) Argue that $AA \subsetneq ACA$ by (1) showing that if $w \in AA$, then $w \in ACA$, and (2) giving an example of a string w which is in ACA but not in AA.

   (g) Argue that $AB = ACB$ by showing that each string w is a member of AB if and only if w is also a member of ACB.