Directions: You may work to solve these problems in groups, but all written work must be your own. See "Guidelines and advice" on the course webpage for more information.

1. *The halting problem.* Alan Turing proved the halting problem is undecidable. In this problem, we develop a proof. Suppose that there is a program called ⟨HALT⟩ that, given the computer source code to a program P_n and an input x, decides whether or not P_n will eventually terminate when it is run on x. (Note that ⟨HALT⟩ must itself always terminate with a correct yes or no answer, depending on the behavior of P_n on x.)

   Let us define a new program ⟨D⟩ that uses ⟨HALT⟩ as a subroutine. Here is a description of the code for ⟨D⟩:

   ```
   D:
     Given the source code S to a program P_n:
         Call ⟨HALT⟩ on P_n and input S

         If ⟨HALT⟩ reports that P_n terminates on S:
             loop forever
         Else:
             terminate
   ```

   What will happen if we run ⟨D⟩ on its own source code? What can you conclude?

2. It is estimated that there are 10^{80} atoms in the observable universe. If we start with the emptyset $\varnothing$ and apply the power set operation, then we get $\{\varnothing\}$, a set which has size 1. If we iterate the power set operation a second time, then we get $\{\varnothing, \{\varnothing\}\}$, which has size 2. How many times do we need to iterate the power set operation before we get a set whose size is larger than the number of atoms in the observable universe?

3. Suppose that we roll a fair, 6-sided die 3 times.

   (a) What is the sample space Ω?

   (b) What is the probability that all three rolls have the same parity (i.e. all rolls are even or all rolls are odd)?

   (c) What is the probability that three distinct values are rolled?

   (d) What is the probability that at least one of the rolls is a 6?

   (e) What is the probability that the sum of the three rolls is 10?