Math375 **Homework 6** **Due Sep. 28, 2021**

Directions: You may work to solve these problems in groups, but all written work must be your own. See "Guidelines and advice" on the course webpage for more information.

1. Let $A = \{1, 2\}$, let $B = \{2, 3, (4, 5)\}$, and let $C = \{\emptyset, \{1, 2\}, \{2, 1\}\}$, $D = \{2, 3, (5, 4)\}$.

   (a) Determine $|A|$, $|B|$, and $|C|$.

   (b) Determine the sets $A \times B$ and $A \times C$.

   (c) True or False: $C \subseteq \mathcal{P}(A)$

   (d) True or False: $A \subseteq \mathcal{P}(C)$

   (e) Determine the set $B \triangle D$.

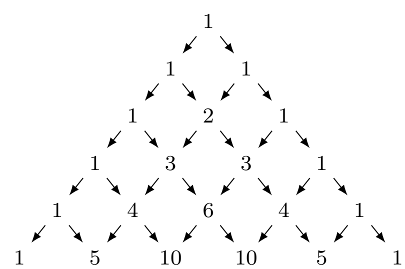
2. *Pascal's Triangle.* Let $[n] = \{1, 2, 3, \ldots, n\}$.

   (a) How many k-element subsets of $[n]$ *do not* contain n?

   (b) How many k-element subsets of $[n]$ *do* contain n?

   (c) Apply the rule of sum to explain Pascal's Formula: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

   (d) Pascal's Triangle is a triangular array where each row starts and ends with a 1, and every other entry is obtained by adding the nearest two entries in the previous row.



   Describe the relationship between Pascal's Triangle and Pascal's formula.

3. A *bitstring* is an ordered list of zeros and ones; for example, 0110 and 10100 are bitstrings of lengths 4 and 5, respectively. As a special case, we use ε to denote the empty bitstring, which has length 0.

   (a) Show that the set of all bitstrings of finite length is countable.

   (b) Is the set of all bitstrings of infinite length countable? Justify your answer.

4. The Cartesian product can be extended to more than two factors. For example, $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$, or $\mathbb{N}^3$ for short, is the set $\{(a_1, a_2, a_3) \colon a_1 \in \mathbb{N} \text{ and } a_2 \in \mathbb{N} \text{ and } a_3 \in \mathbb{N}\}$. Show that $\mathbb{N}^3$ is countable.