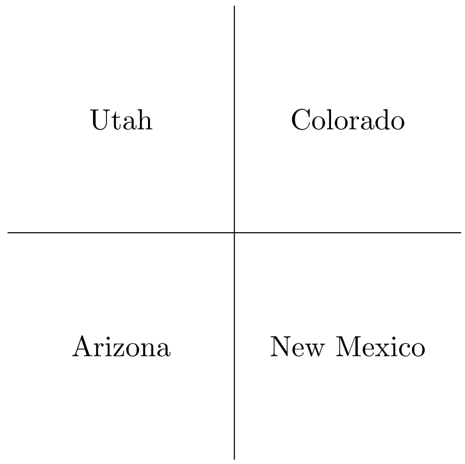
Directions: You may work to solve these problems in groups, but all written work must be your own. Show all work.

1. In the United States, the states of Utah,. Colorado, Arizona, and New Mexico all meet at a single point called the four corners.



A mapmaker has 8 colors available for a map of these four states. States that share a border of positive length must be colored differently. For example, Utah and Colorado must have distinct colors, but Utah and New Mexico (whose borders only intersect at one point) are allowed to have the same color. We wish to count the number of ways to color these 4 states.

   (a) Explain the error in the following counting scheme.

      i. Color Utah. [8 options]

      ii. Next, color Colorado with a color different from Utah. [7 options]

      iii. Next, color New Mexico with a color different from Colorado. [7 options]

      iv. Finally, color Arizona with a color different from Utah and New Mexico. [6 options].

   By the rule of product, there are $8 \cdot 7 \cdot 7 \cdot 6$, or $2,352$ ways to color these four states.

   (b) What is the correct number of ways the map can be colored?

2. How many was are there to arrange the letters of 'ECCENTRIC':

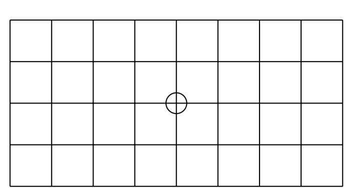
   (a) with no additional restrictions?

   (b) beginning *and* ending with a C?

   (c) beginning *or* ending with a C (or both)? (Note: CECENTRIC is allowed.)

   (d) with all three C's consecutive?

3. Lattice paths from $(0, 0)$ to $(8, 4)$.



(a) How many lattice paths are there from $(0,0)$ to $(8,4)$ in which each step increases one of the coordinates by 1?

(b) Suppose there is a deadly dragon that lives at the center $(4,2)$. How many lattice paths from $(0,0)$ to $(8,4)$ avoid the dragon?

4. How many ways are there to partition the integers $\{1,2,3,\ldots,2n\}$ into pairs? For example, when $n=2$, there are 3 ways: $\{\{1,2\},\{3,4\}\},\{\{1,3\},\{2,4\}\},\{\{1,4\},\{2,3\}\}$. Note that the order of the pairs is not important (so $\{\{3,4\},\{1,2\}\}$ is the same is $\{\{1,2\},\{3,4\}\}$ and the order of within pairs is unimportant (so $\{\{2,1\},\{3,4\}\}$ is the same as $\{\{1,2\},\{3,4\}\}$).

   Hint: There are several ways to solve this. Here is one nice way. Count the number of permutations of $\{1,2\ldots,2n\}$ of length $2n$ in two different ways: one directly, and the other using the rule of product (where one stage involves the unknown quantity).