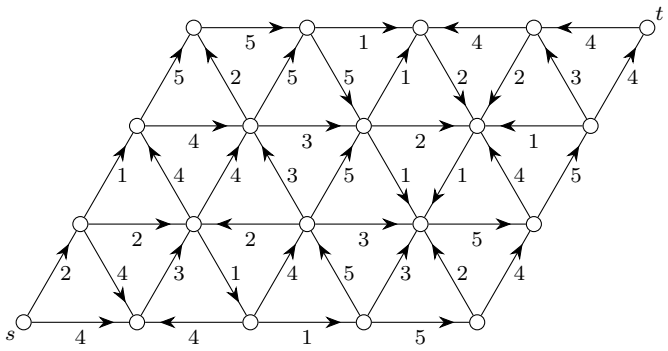
Directions: You may work to solve these problems in groups, but all written work must be your own. See "Guidelines and advice" on the course webpage for more information.

1. A computer network has 12 nodes. Since the computers are far apart, it is very expensive to add a communications link between two nodes. The network must be robust in that the network must remain connected even if up to 2 communication links fail. What is the minimum number of communications links needed?

2. Prove that if G is a planar graph, then G has a vertex of degree at most 5.

3. Find a max. flow and a min. cut in the following network.



4. In a graph G, a *matching* is a set of edges M such that every vertex is incident to at most one edge in S. A *perfect matching* is a matching M in which every vertex is incident to exactly one edge in M. Find all perfect matchings in the cube. How many are there?