Directions: You may work to solve these problems in groups, but all written work must be your own. See "Guidelines and advice" on the course webpage for more information.

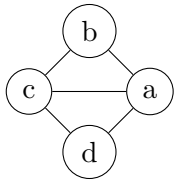
1. The *complement* of a graph G, denoted $\overline{G}$, is the graph with vertex set $V(G)$ with u and v adjacent in $\overline{G}$ if and only if u and v are not adjacent in G.

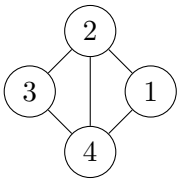
   (a) Let G be a graph with n vertices and m edges. How many edges does $\overline{G}$ have?

   (b) Prove that for every graph G, either G or $\overline{G}$ is connected. (Hint: if G is disconnected, then use a lemma from class that partitions $V(G)$ into two nonempty sets with a certain property.)

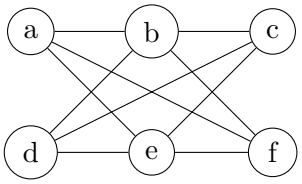
2. Decide whether the following pairs of graphs are isomorphic. If they are isomorphic, give the function that establishes the isomorphism. If not, explain why.

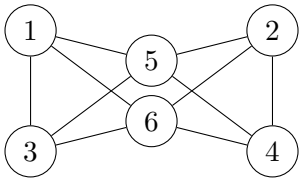
(a)



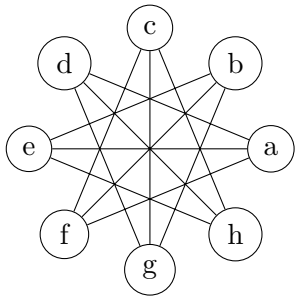


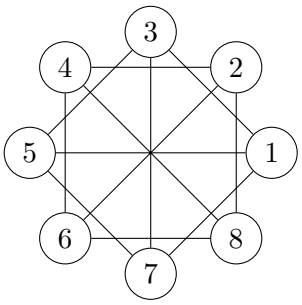
(b)





(c)





3. Show that when any edge is removed from K_5, the resulting subgraph is planar. Is this true for $K_{3,3}$?

4. Let G be a connected planar graph (without loops or parallel edges). One way of embedding G in the plane creates 53 regions, each of which has at least five edges on its boundary. Prove that G has at least 82 vertices.