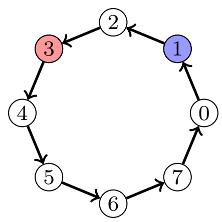
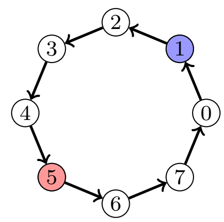
Directions: You may work to solve these problems in groups, but all written work must be your own. See "Guidelines and advice" on the course webpage for more information.

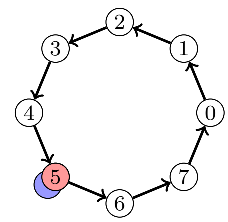
1. Two players, Advancer and Chooser, play a game on the vertices of an octagon, labeled with entries in $\{0, \ldots, 7\}$. Advancer is blindfolded and never sees the state of the game. Initially, Chooser places a red token and a blue token on the octagon. In each round, Advancer picks an integer k and Chooser decides whether to move the red token or the blue token counter-clockwise by k vertices. Advancer wins if, at any point in the game, the tokens occupy the same vertex.



(a) Chooser places the tokens to start the game. Advancer picks $k = 2$. Chooser must move the red token forward or else Advancer wins immediately.



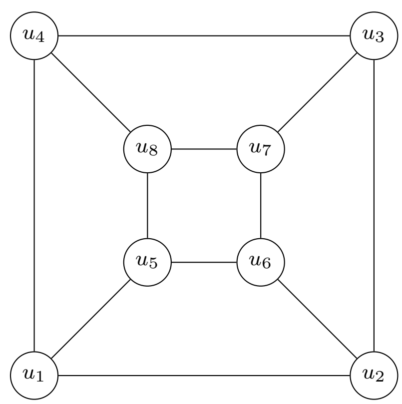
(b) In the next round, Advancer can force a win by picking $k = 4$.



(c) With the tokens on the same vertex, Advancer has won.

Show that Advancer has a strategy to win in at most 7 rounds. (Hints: model the game with an NFA with 4 states, one for each of the possible nonzero gaps between the two tokens. When converting to a DFA, try to construct only the relevant parts of the machine.)

2. Let G be the following graph.



   (a) Find two disjoint 4-cycles in G.

   (b) Find a 6-cycle in G.

   (c) Does G contain an 8-cycle? If so, describe one. If not, explain why not.

   (d) Show how to color the vertices red and blue so that no edge has two endpoints with the same color.

   (e) Does G contain a 5-cycle? If so, describe one. If not, explain why not.