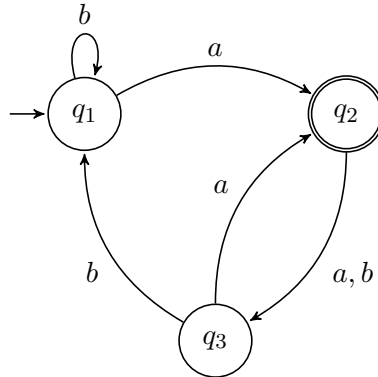**Directions:** You may work to solve these problems in groups, but all written work must be your own. See "Guidelines and advice" on the course webpage for more information.

1. Let $\Sigma = \{a, b\}$ and let M be the automaton pictured below (from Sipser 1.1).

    (a) Give a simple English description for $L(M)$.

    (b) Two automata are *equivalent* if their languages are equal. The DFA above has 3 states but is equivalent to a DFA with only 2 states. Construct a DFA with 2 states that is equivalent to M.

2. Let $\Sigma = \{0, 1\}$. Give state diagrams of DFAs for the following languages.

    (a) $\{w \mid w$ begins with a 1 and ends with a 0$\}$.

    (b) $\{w \mid w$ has an even number of 1s or contains the substring 101$\}$

3. A *palindrome* is a string that reads the same forwards and backwards. Let $\Sigma = \{a, b\}$, and let $A = \{w \in \Sigma^* : w$ is a palindrome$\}$. For example, $\lambda, a, aba, abbabaababba \in A$ but $abb, aaba \notin A$. Show that A is not a regular language. Your argument should primarily use English sentences, with proper grammar, spelling, and punctuation.