Name: _____

Directions: Show all work. Answers without work generally do not earn points.

1. **[4 parts, 3 points each]** Let $\Sigma = \{0, 1, 2\}$. Define the following languages.

   $A = \{w \mid$ the symbols of w are in sorted order (non-decreasing from left to right)$\}$
   $B = \{w \mid w$ has more 1's than 0's$\}$
   $C = \{w \mid$ the symbols of w can be split into two groups whose sums are equal$\}$

   For example: λ and 01201 are both words in C (we can split 01201 into two groups with equal sum by putting both one's in the first group and the remaining symbols in the second group), and $1211 \notin C$ (no split is possible).

   (a) Give an example of a string in $A \cap B \cap C$.

   (b) Give an example of a string in $A - B$.

   (c) True or False: $A \cap B \subseteq C$.

   (d) True or False: $BB = B$.

2. **[4 points]** Let $\Sigma = \{0\}$. Find all languages over Σ that are computable by DFAs with at most 2 states.

3. **[4 parts, 4 points each]** Let $\Sigma = \{a, b\}$. Construct DFAs for the following languages.

(a) $\{w \mid w$ has at least two a's$\}$

(b) $\{w \mid w$ has no pair of consecutive b's$\}$

(c) $\{w \mid$ the length of w is divisible by 3$\}$

(d) $\{w \mid w$ ends with $ab\}$

4. Let $\Sigma = \{a, b\}$ and define the following languages.

$$A_1 = \{(ab)^n \mid n \geq 0\} = \{\lambda, ab, abab, ababab, \ldots\}$$
$$A_2 = \{w \mid w \text{ has an odd number of } a\text{'s}\}$$

(a) **[5 points]** Give a simplified DFA for A_1.

(b) **[5 points]** Give a simplified DFA for A_2.

(c) **[8 points]** Give a DFA for $A_1 \cup A_2$.

5. **[4 parts, 4 points each]** Let $\Sigma = \{a, b, c\}$. For each string $w \in \Sigma^*$, let $\#a(w)$, $\#b(w)$, and $\#c(w)$ denote the number of a's, b's, and c's in w. Define the following languages.

   $A = \{w \mid w$ contains a consecutive pair of a's$\}$
   $B = \{w \mid \#b(w) = 1$ and the single b in w is the second to last symbol$\}$
   $C = \{w \mid$ the 3-tuple $(\#a(w), \#b(w), \#c(w))$ contains at least one even integer$\}$

   Give NFAs for the following using at most the specified number of states.

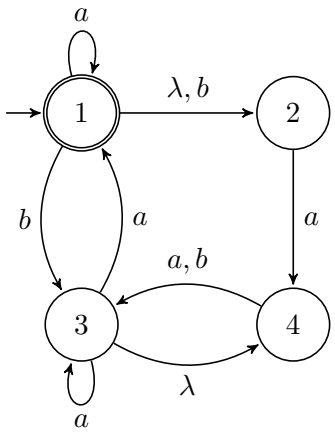
   (a) A, at most 3 states

   (b) B, at most 3 states

   (c) AB, at most 5 states

   (d) C, at most 7 states

6. Let $\Sigma = \{a, b\}$ and let N be the following NFA (below left).



State	a	b
1		
2		
3		
4		

(a) **[8 points]** For each state/input pair, give the set of successor states in the table above.

(b) **[6 points]** Convert N to a DFA.

(c) **[4 points]** Simplify your DFA above. Use your simplified DFA to give a simple description of the language computed by N.

7. **[16 points]** Let $\Sigma = \{a, b\}$ and define the following languages.

$$A = \{w \mid w \text{ starts and ends with an } a\}$$
$$B = \{w \mid w \text{ has an even length}\}$$

Note that $\lambda \notin A$ but $a \in A$. Give a simplified DFA that computes the language BA.

8. **[4 bonus points]** Let $\Sigma = \{0, 1, 2\}$ and recall the language C from problem (1), where

$$C = \{w \mid \text{the symbols of } w \text{ can be split into two groups whose sums are equal}\}.$$

Is the language C regular? Justify your answer.

Scratch Paper