Name: _____

Directions: Show all work. Answers without work generally do not earn points. You mean leave numerical answers in terms of factorials and binomial/multinomial coefficients.

1. **[6 parts, 2 points each]** Let $A = \{2, 3, \{2, 3\}\}$, $B = \{\varnothing, 2, \{3\}\}$, and $C = \{(3, 2), (2, 3), \{2, 3\}, \{3, 2\}\}$. For the True or False questions, **write the entire word**.

   (a) Determine the sizes $|A|$, $|B|$, and $|C|$.

   (b) True or False: $\{3, 2\} \in C - A$

   (c) True or False: $(3, 2) \in C - A$

   (d) Determine $A \cap B$.

   (e) Determine $A \cup B$.

   (f) Determine $\mathcal{P}(B - A)$.

2. **[2 parts, 2 points each]** A set A of integers is *closed under addition* if $x + y \in A$ whenever $x \in A$ and $y \in A$.

   (a) Give two examples of an infinite set of integers that is closed under addition.

   (b) Which finite sets of integers, if any, are closed under addition? Explain.

3. **[4 parts, 3 points each]** Let A, B, and C be sets. Express the following sets as concisely as possible using standard set operations. For example, "The set of all elements that are in A or B" is $A \cup B$.

   (a) The set of all elements that are in B but not A.

   (b) The set of all ordered pairs of elements in C.

   (c) The set of all elements that belong to exactly one of A and C.

   (d) The set of all sets X such that $X \subseteq A \cup B$, $X \cap A \neq \varnothing$, and $X \cap B \neq \varnothing$.

4. **[2 parts, 3 points each]** Let A be a set of size n.

   (a) Determine the size of $\mathcal{P}(A \times A)$.

   (b) Determine the size of $\mathcal{P}(A) \times \mathcal{P}(A)$.

5. **[5 points]** Are there any sets whose size is larger than the set of real numbers $\mathbb{R}$? If yes, then give an example. If no, then explain why not.

6. **[5 points]** Identify a significant consequence of Cantor's argument that the real numbers are uncountable in the field of computer science.

7. **[6 points]** Let A and B be countable sets. Argue that $A \times B$ is countable.

8. **[3 parts, 4 points each]** A class contains 8 men and 10 women. A group of 6 students is chosen at random.

   (a) What is the probability that all students in the chosen group are men?

   (b) What is the probability that all students in the chosen group are women?

   (c) What is the probability that the chosen group has at least one man and at least one woman?

9. **[4 points]** Two different cards are drawn at random from a deck of 12 cards, each labeled with an integer in $\{1, \ldots, 12\}$. What is the probability that the difference between the values on the chosen cards is at most 3?

10. Suppose that a pair of dice are rolled. Let A be the event that the two rolled values are the same, let B be the event that the sum is in $\{6, 7, 8\}$, and let C be the event that both values rolled are at least 4.

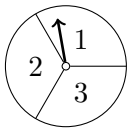
    (a) **[3 points]** Give the sample space Ω.

    (b) **[6 points]** Find $\Pr(A)$, $\Pr(B)$, and $\Pr(C)$.

    (c) **[6 points]** Find $\Pr(A \cap B)$, $\Pr(B \cap C)$, and $\Pr(C \cap A)$.

    (d) **[3 points]** For each of the pairs of events $\{A, B\}$, $\{B, C\}$, and $\{C, A\}$, determine whether the pair is independent, positively correlated, or negatively correlated.

11.



A spinner has three equal regions, labeled 1, 2, and 3. Each spin is equally likely to stop in each of the three regions.

(a) **[6 points]** A contestant executes 3 spins. What is the probability that each region is the result of one spin?

(b) **[6 points]** A contestant executes n spins. what is the probability that each region is the result of at least one spin? (Note: your answer should be a formula involving n which agrees with part (a) when $n = 3$.)

(c) **[4 points]** A contestant executes n spins. What is the probability that the spins appear in order, with all region 1 spins happening before all region 2 spins, and all region 2 spins happening before all region 3 spins?

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