Name: _____

Directions: Show all work. Answers without work generally do not earn points. Unless stated otherwise, answers may be left in terms of factorials and binomial coefficients.

1. **[4 points]** How many subsets of $\{1, 2, \ldots, 14\}$ have size 10? Express your answer as a concrete, simplified number.

2. **[3 parts, 4 points each]** A restaurant offers a combo meal; customers choose one of 5 sandwiches, one of 3 sides, and one of 7 beverages. Charles and Marie wish to order combo meals.

   (a) How many ways can Charles and Marie order combo meals?

   (b) How many ways can Charles and Marie order so that they avoid placing identical orders? (For example, a situation in which Charles and Marie order the same sandwich, the same beverage, but different sides counts.)

   (c) How many ways are there for Charles and Marie to order combo meals with different sandwiches, different sides, and different beverages?

3. **[4 parts, 4 points each]** A 4-digit ATM pin is a list of 4 digits, like 0000 and 5398. How many 4-digit ATM pins:

   (a) are there in total?

   (b) do not contain the digit 3? (So 1425 counts but 8322 does not.)

   (c) contain all distinct digits? (So 5398 counts but 5395 does not.)

   (d) contain at least one even digit and and least one odd digit? (So 3011 counts but 0284 and 5555 do not.)

4. **[4 parts, 4 points each]** How many ways are there to arrange the letters in CORROBORATION:

   (a) with no additional restrictions?

   (b) with all four O's appearing consecutively? (So BROOOORCTAIRN counts but ROOBOORCTAIRN does not.)

   (c) with the O's, the A, and the I appearing before all other letters? (So OIOOAORCBRTNR counts but OIOOAROCBRTNR does not.)

   (d) with all O's appearing before all R's? (So COOBOONRRRTAI counts but CORBOONRORTAI does not.)

5. **[5 parts, 4 points each]** A non-standard card deck has 6 suits (called A, B, C, D, E, and F) and 9 ranks (1 through 9). The deck has one card for each suit/rank pair, for a total of 54 cards. When a variant of poker is played with this deck, a *hand* consists of a set of 7 cards. For example, {2A, 3A, 1B, 8B, 2D, 5E, 9F} is a hand. How many hands:

   (a) are there in total?

   (b) have all 7 cards in the same suit?

   (c) contain no cards of suit A and no cards of rank 1?

   (d) have no two cards with the same rank?

   (e) have a majority (i.e. more than half) of cards belonging to suit A? Express your answer using Sigma (Σ) summation notation.

6. **[4 parts, 4 points each]** A pet store offers 6 types of community fish: danios, guppies, swordtails, platies, rasboras, and tetras. Determine the number of ways to purchase:

   (a) 3 fish, with all fish of distinct types? (So "1 guppy, 1 tetra, and 1 platy" counts, but "2 swordtails and 1 danio" does not.)

   (b) 3 fish, with no additional restrictions? (So "3 tetras" counts.)

   (c) 15 fish, with at least one fish of each available type?

   (d) at least 10 fish and at most 20 fish?

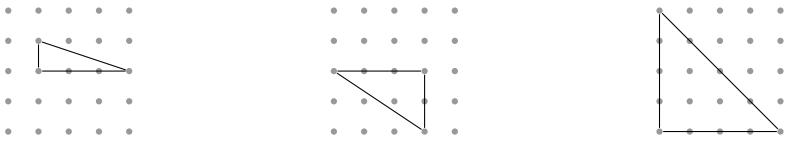
7. **[3 parts, 4 points each]** Determine the following coefficients.

   (a) The coefficient of x^4y^2 in $(x+y)^6$.

   (b) The coefficient of x^8 in $(x+2)^{20}$.

   (c) The coefficient of $w^3x^6y^2z$ in $(w+x+y+z)^{12}$.

8. **[4 points]** An *axis-aligned right triangle* is a triangle that has a horizontal leg and a vertical leg meeting at an angle of 90 degrees. How many axis-aligned right triangles can be formed whose vertices belong to a set of 25 points arranged in a 5×5 grid? Examples follow.



Scratch Paper