

Name: Solutions

Directions: Show all work. No credit for answers without work.

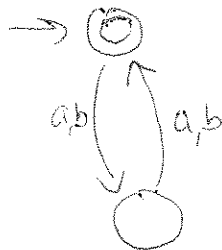
1. Let $\Sigma = \{a, b\}$. Define

$$A_1 = \{w \mid w \text{ has even length}\}$$

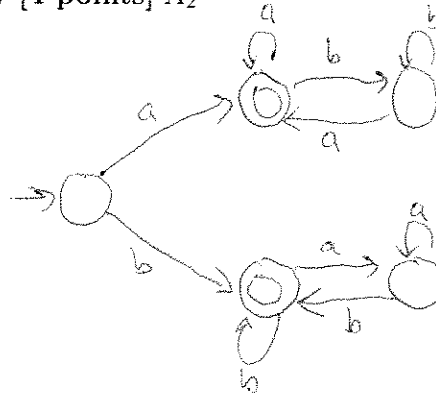
$$A_2 = \{w \mid w \text{ starts and ends with the same symbol}\}$$

Give Deterministic Finite Automata (DFA's) that compute the following languages.

(a) [3 points] A_1



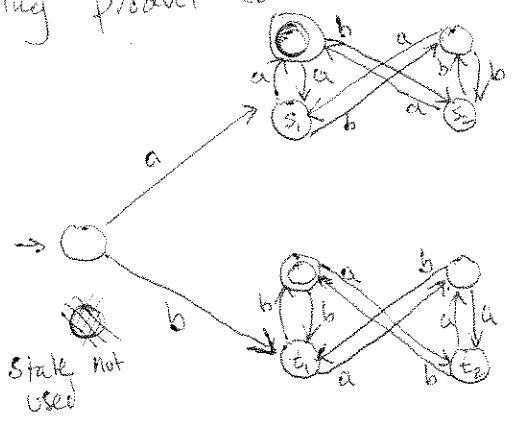
(b) [4 points] A_2



2. [3 points] With A_1 and A_2 as in problem (1), give a DFA for $A_1 \cap A_2$.

①

Using product construction:



②

To simplify, note that we can combine states s_1 and s_2 , because they are both reject states and, no matter if the next input symbol is "a" or "b", the next state is the same starting from s_1 or s_2 . Similarly, we can combine t_1 and t_2 :

③

From ②, redrawn:

