Name: ______________________________

Directions: Show all work. No credit for answers without work.

1. **[5 parts, 2 points each]** Let $\Sigma = \{a, b, c\}$; we define the following languages:

$$F = \{w\colon \text{ the number of } a\text{'s equals the number of } b\text{'s}\}$$
$$G = \{w\colon \text{ the number of } b\text{'s equals the number of } c\text{'s}\}$$
$$H = \{w\colon \text{ all } a\text{'s in } w \text{ appear before all } c\text{'s}\}$$

(a) Give an example of a word $w \in F - G$.

(b) Give two examples of words in $F \cap G$.

(c) True or False: $FF = F$. If True, give an argument justifying your claim. If False, give an example of a word w that belongs to exactly one of the languages in $\{FF, F\}$ and is omitted from the other.

(d) True or False: $FH = H$. If True, give an argument justifying your claim. If False, give an example of a word w that belongs to exactly one of the languages in $\{FH, H\}$ and is omitted from the other.

(e) True or False: $F \cup G \subseteq FG$. If True, give an argument justifying your claim. If False, give an example of a word w that belongs to $F \cup G$ but does not belong to FG.