Name: ______________________________

Directions: Show all work. No credit for answers without work.

1. **[4 parts, 1 point each]** Let $A = \{1, 2, 3\}$, let $B = \{3, 4\}$, and let $C = \varnothing$.

   (a) Determine the sets $A \times B$ and $A \times C$.

   (b) True or False (write the whole word): $(A - B) \subseteq (A - B)^2$.

   (c) Give two examples of elements in $\mathcal{P}(A) \times \mathcal{P}(B)$.

   (d) Give two examples of elements in $\mathcal{P}(A \times B)$.

2. **[2 parts, 1 point each]** Express the following statements using concise mathematical notation. For example, the statement "The set A is a member of the set B" may be expressed as "$A \in B$".

   (a) "Every element in A is also an element in B."

   (b) "Every subset of B and every subset of C is a member of the set A."

3. **[2 points]** An infinite bitstring is *periodic* if it consists of repeated copies of a finite bitstring. For example, $0000\cdots$ consists of repeated copies of 0, and $011011011\cdots$ consists of repeated copies of 011. Let A be the set of periodic infinite bitstrings. Is A countable? Justify your answer.

4. **[2 points]** A sequence $n_1, n_2, n_3, \ldots$ of positive integers is *increasing* if $n_1 < n_2 < \cdots$. Let B be the set of increasing sequences of positive integers. Is B countable? Justify your answer.