

Name: Solutions**Directions:** Show all work. No credit for answers without work.1. [4 parts, 1 point each] Let $A = \{1, 2, 3\}$, let $B = \{3, 4\}$, and let $C = \emptyset$.(a) Determine the sets $A \times B$ and $A \times C$.

$$A \times B = \{(1, 3), (2, 3), (3, 3), (1, 4), (2, 4), (3, 4)\}$$

$$A \times C = \emptyset$$

(b) True or False (write the whole word): $(A - B) \subseteq (A - B)^2$.

False. The integer 1 is a member of $A - B$ but every element in $(A - B)^2$ (or $(A - B) \times (A - B)$) is an ordered pair

(c) Give two examples of elements in $\mathcal{P}(A) \times \mathcal{P}(B)$: Pairs of the form (subset of A , subset of B).

Many examples: (\emptyset, \emptyset) , (A, B) , $(\{1, 3\}, \{3\})$, $(\{1\}, \{3, 4\})$

(d) Give two examples of elements in $\mathcal{P}(A \times B)$: Subsets of $A \times B$.

For instance: \emptyset , $\{(1, 3), (3, 3)\}$, $\{(1, 4), (2, 4), (3, 4)\}$, $A \times B$,

2. [2 parts, 1 point each] Express the following statements using concise mathematical notation. For example, the statement "The set A is a member of the set B " may be expressed as " $A \in B$ ".(a) "Every element in A is also an element in B ."

$$A \subseteq B$$

(b) "Every subset of B and every subset of C is a member of the set A ."

$$\mathcal{P}(B) \cup \mathcal{P}(C) \subseteq A$$

3. [2 points] An infinite bitstring is *periodic* if it consists of repeated copies of a finite bitstring. For example, $0000\dots$ consists of repeated copies of 0, and $011011011\dots$ consists of repeated copies of 011. Let A be the set of periodic infinite bitstrings. Is A countable? Justify your answer.

Yes, A is countable. Begin with an enumeration of all finite, non-empty

bitstrings: $0, 1, 00, 01, 10, 11, 000, 001, 010, \dots$ (We can enumerate finite bitstrings by their lengths, as in HW5.)

Then

Next, replace each finite bitstring^x with the periodic bitstring $xxxx\dots$.

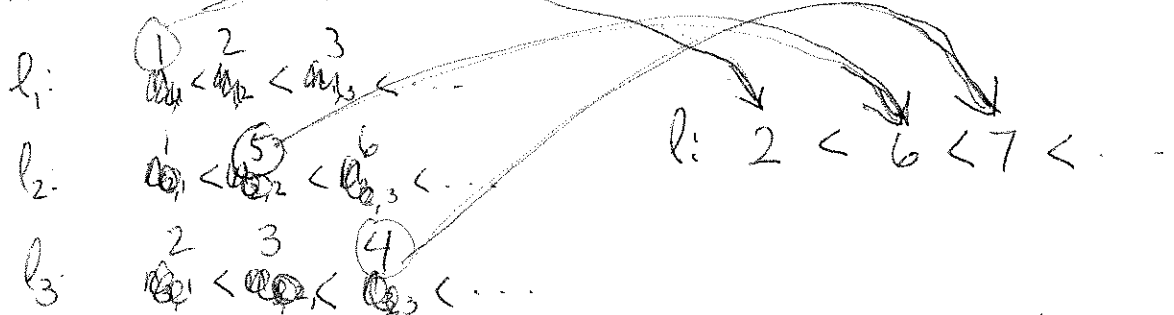
So, $0, 1, 00, 01, 10, 11, \dots$ becomes $000000\dots, 111111\dots, \cancel{000000\dots}, 010101\dots, 101010\dots, \cancel{111111\dots}, \dots$

Finally, cross out repeated bitstrings

4. [2 points] A sequence n_1, n_2, n_3, \dots of positive integers is *increasing* if $n_1 < n_2 < \dots$. Let B be the set of increasing sequences of positive integers. Is B countable? Justify your answer.

No. B is uncountable. We modify Cantor's Diagonalization Argument.

Suppose for a contradiction that such a list ^{l_1, l_2, l_3, \dots} of all such sequences exists:



Make a new list l by ~~setting~~ ^{the i th} entry to be an integer larger than earlier entries in l but still different from the i th entry in l_i . Since l is an increasing list of positive integers and l does not appear in l_1, l_2, \dots , we have a contradiction.