Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[6 parts, 1 point each]** Let $A = \{3, 4, 1, \{2, 1\}\}$, $B = \{\varnothing, \{1\}, \{2\}\}$, and $C = \{1, 2\}$.

   (a) Determine the sizes of A, B, and C.

   $|A| = 4$ $|C| = 2$

   $|B| = 3$

   (b) Determine the set $A - C$.

   $A - C = \boxed{\{3, 4, \{2, 1\}\}}$

   (c) True or False (write entire word): $\{1, 2\} \in A$.

   $\boxed{\text{True}}$ The set $\{1,2\}$ is equal to $\{2,1\}$, and this is one of the elements in A.

   (d) True or False (write entire word): $\{\varnothing\} \in B$.

   $\boxed{\text{False}}$ Although $\varnothing \in B$, the sets $\varnothing$ and $\{\varnothing\}$ are not equal.

   (e) True or False (write entire word): $\{1\} \in \mathcal{P}(B)$.

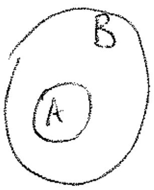
   $\boxed{\text{False}}$ The statement $\{1\} \in \mathcal{P}(B)$ means that $\{1\} \subseteq B$. Since $1 \in \{1\}$ but $1 \notin B$, we have $\{1\} \not\subseteq B$.

   (f) True or False (write entire word): $B \subseteq \mathcal{P}(C)$

   $\boxed{\text{True}}$ The statement $B \subseteq \mathcal{P}(C)$ means that every element in B is a subset in C. This is the case.

2. **[2 points]** Suppose that $A \subseteq B$, meaning that A is a subset of B. Describe the relationship between $\mathcal{P}(B - A)$ and $\mathcal{P}(B) - \mathcal{P}(A)$. Are these sets always equal? Is one always a subset of the other? Explain your answer. Hint: it may help to draw a picture.

These sets are $\boxed{\text{not always equal}}$ and it is not the case that one is always contained in the other.



Note that $\mathcal{P}(B-A)$ consists of all subsets of $B-A$. The set $\mathcal{P}(B)-\mathcal{P}(A)$ consists of subsets of B that are not also subsets of A; these are the subsets of B that contain at least one element of $B-A$. Most subsets of $B-A$ will contain an element of $B-A$; the exception is the empty set $\emptyset$. Indeed, $\emptyset \in \mathcal{P}(B-A)$ but $\emptyset \notin \mathcal{P}(B)-\mathcal{P}(A)$. However, $\boxed{\mathcal{P}(B-A) - \{\emptyset\} \subseteq \mathcal{P}(B)-\mathcal{P}(A)}$.

3. **[2 points]** Give an example of a set A of size at least 2 such that $A \subseteq \mathcal{P}(A)$. (Partial credit for giving a smaller set A that satisfies $A \subseteq \mathcal{P}(A)$.)

Note that $A \subseteq \mathcal{P}(A)$ means that every element in A is also a subset of A.

Take $A = \{\emptyset, \{\emptyset\}\}$. This works because $\emptyset \subseteq A$ and $\{\emptyset\} \subseteq A$ (since $\emptyset \in A$).