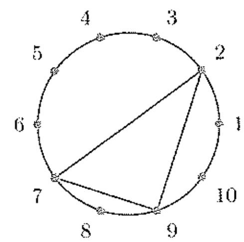
Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[1 point]** Suppose that a set of 10 points are equally spaced around a circle. How many triangles can be formed whose vertices are among the equally spaced points? One such triangle is shown below. Express your answer as a simplified, concrete number.



$$\binom{10}{3} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{3! \cdot \cancel{7!}} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 10 \cdot 3 \cdot 4 = \boxed{120}$$

2. Recall that a standard deck of cards has 4 suits (clubs, hearts, diamonds, spades) and 13 ranks (ace, 2 through 10, jack, queen, king), and there are 52 cards in total, one for each suit/rank pair. A poker hand is a set of five cards.

   (a) **[2 points]** How many poker hands have exactly two clubs?

Stage 1: Choose ranks for two clubs $n_1 = \binom{13}{2}$

Stage 2: Choose 3 non-clubs $n_2 = \binom{39}{3}$

$$\text{Total } \# = \boxed{\binom{13}{2} \cdot \binom{39}{3}} = 78 \cdot 9139 = \boxed{712{,}842}$$

   (b) **[2 points]** How many poker hands have an even number of clubs? You may leave your answer as a summation.

Hands with 0 clubs: $\binom{13}{0} \cdot \binom{39}{5}$

Hands with 2 clubs: $\binom{13}{2} \cdot \binom{39}{3}$

Hands with 4 clubs: $\binom{13}{4} \cdot \binom{39}{1}$

$$\boxed{\binom{13}{0} \cdot \binom{39}{5} + \binom{13}{2} \cdot \binom{39}{3} + \binom{13}{4} \cdot \binom{39}{1}} = \boxed{1{,}316{,}484}$$

3. How many ways are there to arrange the letters in the word '~~ATTRACTION~~':

A-2 R-1 I-1
T-3 C-1 O-1
N-1

(a) **[2 points]** with no additional restrictions?

$$\frac{(10)!}{(2)!\,(3)!\,(1)!\,(1)!\,(1)!\,(1)!\,(1)!} = \boxed{\frac{10!}{2!\cdot 3!}} = \boxed{302,400}$$

(b) **[2 points]** with no consecutive T's? (So AATRCITONT counts but ATTRACTION and ATTTRACION do not.)

Stage 1: Arrange AARCION, no restrictions $\quad n_1 = \frac{7!}{2!}$

Stage 2: Insert 3 T's into 8 distinct spaces $\uparrow A \uparrow A \uparrow R \uparrow C \uparrow I \uparrow O \uparrow N \uparrow$ $\quad n_2 = \binom{8}{3}$

$$\text{So total} = \frac{7!}{2!}\cdot\binom{8}{3} = \boxed{\frac{7!}{2}\cdot\binom{8}{3}} = \boxed{141,120}$$

(c) **[1 point]** with no consecutive repeated letters? (So TATATCIONR counts but TAATCTIONR and TATTACIONR do not.)

All Arrangements with no consecutive T's

arrangements with no consecutive T's but A's together

no consec. repeated letters

From all ways in part (b), subtract the # of arrangements with ~~one consecutive~~ consecutive A's; to compute, we glue A's together.

Stage 1: Arrange ⟨AA⟩RCION $\quad n_1 = 6!$

Stage 2: Insert T's into distinct spaces $\uparrow \langle AA\rangle \uparrow R \uparrow C \uparrow I \uparrow O \uparrow N \uparrow$ $\quad n_2 = \binom{7}{3}$

$$\text{So total} = \boxed{\frac{7!}{2}\cdot\binom{8}{3} - 6!\binom{7}{3}} = \boxed{115,920}$$