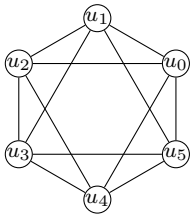
Name: _____

Directions: Show all work. No credit for answers without work.

1. **[2 parts, 2 points each]** Let G be the following graph.



   (a) The 3-vertex cycle, or triangle, is denoted C_3. Find two disjoint copies C_3 in G.

   (b) Which cycles are subgraphs of G?

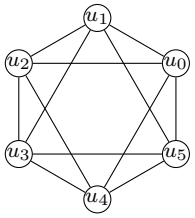
2. **[3 points]** Let G be the graph whose vertex set is the set of all 3-digit ATM pin numbers, where two pin numbers are adjacent if and only if the pins differ in exactly one digit. For example, 601 and 201 are adjacent, 000 and 050 are adjacent, but 123 and 234 are not adjacent. How many edges does G have?

3. Let G be the graph where $V(G) = \{0, 1, 2, \ldots, 9\}$ and $E(G) = \{uv \mid u + v \text{ is a perfect square}\}$. For example, 7 and 9 are adjacent because $7 + 9$ equals the perfect square 16.

   (a) **[2 points]** Draw a copy of G.

   (b) **[1 point]** Does G have a path from 2 to 6? If so, then give such a path. Otherwise, explain why not.

4. **[1 Bonus Point]** Recall the graph G from Question 1.



   What is the minimum number of edges that must be deleted from G to obtain a subgraph that contains no triangles? Prove your answer is correct.