Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[2 parts, 2 points each]** A florist offers 5 different types of vase and 12 different types of flower. To order a flower arrangement, a customer specifies the desired type of vase and the desired type of flower.

   (a) How many ways are there to order a flower arrangement?

Rule of Product.

Stage 1: Choose vase $(n_1 = 5)$

Stage 2: Choose flower $(n_2 = 12)$

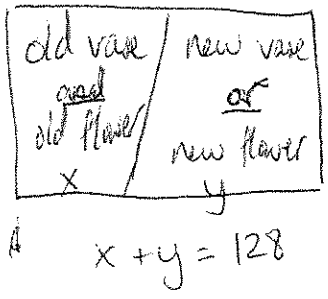
Total $= n_1 \cdot n_2 = 5 \cdot 12 = \boxed{60}$

   (b) One day, the florist decides to add 3 new vase types and 4 new flower types. How many ways are there to order flower arrangements that ~~use~~ have a new vase type or a new flower type (or both)?

With the new vases and new flowers, there are

$(5+3) \cdot (12+4) = 8 \cdot 16 = 128$

options. Of these, 60 use old vases and old flowers



So # arrangements with new vase or new flower (or both) is

$128 - 60 = \boxed{68}$.

2. **[3 parts, 2 points each]** Determine the number of ways that integers in $\{1, 2, 3, 4, 5, 6\}$ can be arranged in some order, subject to each of the following.

   (a) No additional restrictions.

$$P(6,6) = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{720}$$

   (b) All odd numbers come before all even numbers? (For example, 3 1 5 6 4 2 counts but 3 1 6 5 4 2 does not.)

Stage 1: Order $\{1,3,5\}$ in first 3 positions $\quad n_1 = 3! = 3 \cdot 2 \cdot 1 = 6$

Stage 2: Order $\{2,4,6\}$ in last 3 positions $\quad n_2 = 3! = 3 \cdot 2 \cdot 1 = 6$

So total # is $3! \cdot 3! = 6^2 = \boxed{36}$.

   (c) The even numbers are consecutive? (For example, 3 2 4 6 5 1 counts but 3 2 4 5 6 1 does not.)

Note: there are 4 patterns that work; using "e" for even and "o" for odd, these are eeeooo, oeeeoo, ooeeeo, cooeee.

Stage 1: Choose a pattern $(n_1 = 4)$

Stage 2: Order $\{2,4,6\}$ and insert into chosen pattern $(n_2 = 3! = 6)$

Stage 3: Order $\{1,3,5\}$ and insert into pattern $(n_3 = 3! = 6)$

So the total # of arrangements is $4 \cdot 6^2 = \boxed{144}$.