Directions: You may work to solve these problems in groups, but all written work must be your own. **Show your work**; See "Guidelines and advice" on the course webpage for more information.

1. Let $\Sigma = \{0, 1\}$, let $A = \bigcup_{k=0}^{2} \Sigma^k$, and let $B = \bigcup_{k=0}^{4} \Sigma^k$.
   (a) List the strings in A. What is $|A|$?
   (b) What is $|B|$?
   (c) Recall that $AB = \{xy \mid x \in A \text{ and } y \in B\}$. Describe the members of AB. What is $|AB|$?

2. Let $\Sigma = \{0, 1\}$. We define languages A, B, and C as follows:

$$A = \{w \in \Sigma^* \colon\ w \text{ contains more zeros than ones}\}$$
$$B = \{w \in \Sigma^* \colon\ w \text{ contains more ones than zeros}\}$$
$$C = \Sigma^*.$$

   (a) Give an example of a string x that belongs to the language A and a string y that *does not* belong to the language A.
   (b) Give a description of the language $A \cup B$.
   (c) Give a description of the language $A \cap B$.
   (d) Give a description of the language $\overline{A \cup B}$.
   (e) Give a description of the language AA.
   (f) Argue that $AA \subsetneq ACA$ by (1) showing that if $w \in AA$, then $w \in ACA$, and (2) giving an example of a string w which is in ACA but not in AA.
   (g) Argue that $AB = ACB$ by showing that each string w is a member of AB if and only if w is also a member of ACB.