Directions: You may work to solve these problems in groups, but all written work must be your own. **Show your work**; See "Guidelines and advice" on the course webpage for more information.

1. A fair coin is flipped n times. In terms of n, determine the probability of the following events.
   (a) All flips are heads.
   (b) At least one flip is tails.
   (c) No consecutive pair of flips is the same (i.e. both heads or both tails).
   (d) There are the same number of heads as tails. (Note: when n is odd, this probability is zero, but what is the probability when n is even?)

2. Suppose that n people each choose a number independently at random from the set $\{1, \ldots, 1000\}$. What is the smallest value of n for which it becomes more likely than not that at least two people choose the same number?

3. Sasha and Thomas each toss a fair coin 3 times.
   (a) For $0 \le i \le 3$, let A_i be the event that i of Sasha's flips are heads (and the other $3 - i$ flips are tails). For each $0 \le i \le 3$, determine $\Pr(A_i)$.
   (b) What is the probability that Sasha and Thomas toss exactly the same number of heads?
   (c) What is the probability that Sasha gets more heads than Thomas?

4. The numbers in the set $\{1, 2, \ldots, 20\}$ are arranged in a random order. What is the probability that there are consecutive perfect squares? (Hint: think about the complementary event.)

5. Suppose that a set R of 5 cells is chosen at random from the 9 cells in a tic-tac-toe board. What is the probability that R contains all cells from some row or column of the board? (Note: the event does not include situations in which R contains a diagonal, unless R also contains a row or column.)