Directions: You may work to solve these problems in groups, but all written work must be your own. **Show your work**; See "Guidelines and advice" on the course webpage for more information.

1. A pair of dice is rolled. Let A be the event that both dice show the same value, let B be the event that the sum is at least 9, and let C be the event that the first die is in the lower half (i.e. is 1, 2, or 3).

   (a) Express the event A as a subset of the sample space Ω.

   (b) Determine $\Pr(A)$, $\Pr(B)$, and $\Pr(C)$.

   (c) Determine $\Pr(A \cap B)$, $\Pr(B \cap C)$, and $\Pr(C \cap A)$.

   (d) For each pair of events $\{A, B\}$, $\{B, C\}$, and $\{C, A\}$, decide whether the events are independent, positively correlated, or negatively correlated.

2. [3.4.2] Joshua draws two ping-pong balls from a bowl of twenty ping-pong balls numbered 1 to 20. Provide a sample space Ω for this experiment if

   (a) the first ball drawn is replaced before the second ball is drawn.

   (b) the first ball drawn is not replaced before the second ball is drawn.

3. [3.4.10] Twenty-five slips of paper, numbered $1, 2, \ldots, 25$ are placed in a box. If Amy draws six of these slips, without replacement, determine the probability of the following. Let A be the event that the second smallest number drawn is 5 and let B be the event that the fourth smallest number drawn is 15.

   (a) Determine $\Pr(A)$.

   (b) Determine $\Pr(B)$.

   (c) Determine $\Pr(A \cap B)$.

   (d) Determine $\Pr(A \cup B)$.

4. [3.5.6] Let Ω be a sample space and let A and B be events. Find a formula for $\Pr(A \triangle B)$ in terms of $\Pr(A)$, $\Pr(B)$, and $\Pr(A \cap B)$.

5. [3.5.9] Juan tosses a fair coin five times. What is the probability that, after each toss, the total number of heads is strictly larger than the total number of tails?