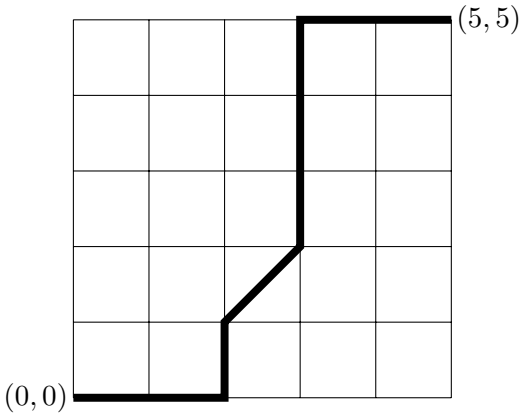
Directions: You may work to solve these problems in groups, but all written work must be your own. **Show your work**; See "Guidelines and advice" on the course webpage for more information.

1. Binomial theorem.
   (a) Find the coefficient of $x^6y^2z^3$ in $(2x - y + 3z)^{11}$.
   (b) Compute $\sum_{k=0}^{n} 2^k \binom{n}{k}$.
   (c) Compute $\sum_{k=0}^{n} \frac{1}{k!(n-k)!}$. Hint: recall the formula for $\binom{n}{k}$. Relate the given sum to one involving binomial coefficients.

2. How many 5-digit ATM pin numbers:
   (a) have distinct digits that increase from left to right? (So 02379 counts, but 02279 and 20458 do not.)
   (b) have digits that are non-decreasing from left to right? (So 02379 and 02279 count, but 20458 does not.)

3. *Solutions to equations.* Count the number of non-negative integral solutions to the following equations.
   (a) $x_1 + x_2 + \cdots + x_6 = 50$
   (b) $x_1 + x_2 + \cdots + x_6 = 50$ where each x_i is at least 4
   (c) $x_1 + x_2 + \cdots + x_6 = 50$ where $x_1 \leq 20$
   (d) $x_1 + x_2 + \cdots + x_6 = 50$ where $1 \leq x_i \leq 30$ for all i.

4. *Lattice paths with diagonal steps.* A *diagonal step* in a lattice path moves 1 unit in the x-direction and 1-unit in the y direction.



   (a) For each k with $0 \leq k \leq 5$, determine the number of lattice paths with diagonal steps from $(0, 0)$ to $(5, 5)$ that have exactly k diagonal steps. (A lattice path from $(0, 0)$ to $(5, 5)$ with 1 diagonal step is displayed above.)
   (b) Add your results from part (a) to determine the total number of lattice paths from $(0, 0)$ to $(5, 5)$ with diagonal steps.
   (c) Using Σ notation, give a summation formula for the number of lattice paths with diagonal steps from $(0, 0)$ to (n, n).