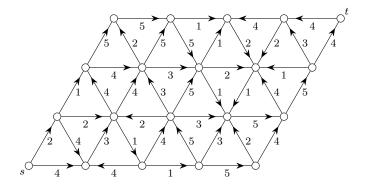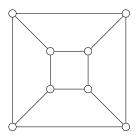**Directions:** You may work to solve these problems in groups, but all written work must be your own. **Show your work**; See "Guidelines and advice" on the course webpage for more information.

1. A computer network has 12 nodes. Since the computers are far apart, it is very expensive to add a communications link between two nodes. The network must be robust in that the network must remain connected even if up to 2 communication links fail. What is the minimum number of communications links needed?

2. Show that when any edge is removed from K_5, the resulting subgraph is planar. Is this true for $K_{3,3}$?

3. Prove that if G is a planar graph, then G has a vertex of degree at most 5.

4. Find a max. flow and a min. cut in the following network.

5. Find all perfect matchings in the cube. How many are there?

6. Note: This problem was removed from HW13 because the corresponding material has not yet been covered in class. It still appears here for studying purposes.

   Find a maximum matching in the following bipartite graph and a set $S \subseteq X$ whose deficiency proves the matching is maximal.