**Directions:** You may work to solve these problems in groups, but all written work must be your own. **Show your work**; See "Guidelines and advice" on the course webpage for more information.

1. Let $G$ be the following graph.
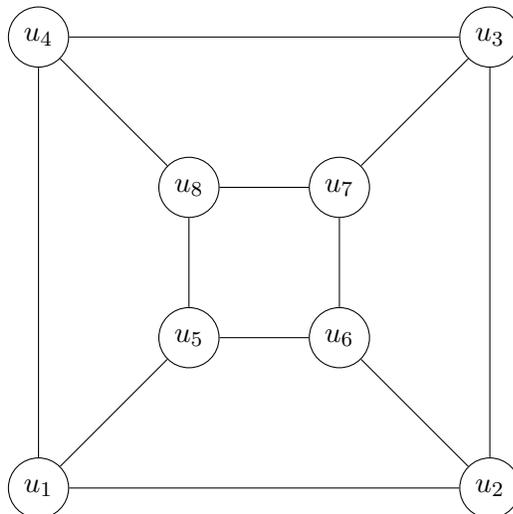


   (a) Find two disjoint 4-cycles in $G$.

   (b) Find a 6-cycle in $G$.

   (c) Does $G$ contain an 8-cycle? If so, describe one. If not, explain why not.

   (d) Show that $G$ is bipartite.

   (e) Does $G$ contain a 5-cycle? If so, describe one. If not, explain why not.

2. Give an example of a graph with 6 vertices, all of which have degree 3. Is there a 7-vertex graph in which all vertices have degree 3?

3. Let $G$ be the graph whose vertex set is the set of all 4-digit ATM pin numbers, where two pin numbers are adjacent if and only if the pins differ in exactly one digit. For example, 6722 and 6022 are adjacent, but 6722 and 6021 are not adjacent. How many edges does $G$ have?

4. Let $G$ be a graph with $n$ vertices, where $n \geq 2$.

   (a) Prove that if $u \in V(G)$, then $0 \leq d(u) \leq n - 1$.

   (b) Prove that $G$ cannot have a vertex of degree 0 and a vertex of degree $n - 1$.

   (c) Prove that $G$ has two vertices of the same degree.