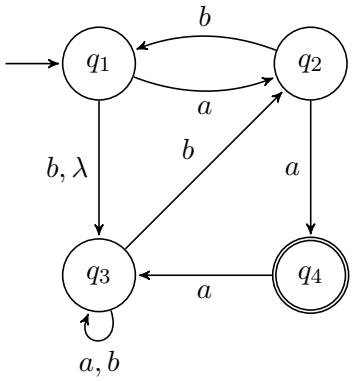
Directions: You may work to solve these problems in groups, but all written work must be your own. **Show your work**; See "Guidelines and advice" on the course webpage for more information.

1. Let $\Sigma = \{0, 1\}$. Let A be the language $\{w \mid w$ is an integer in binary notation and w is divisible by $5\}$. For example, 1010 represents $1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 8 + 2 = 10$, so $1010 \in A$. On the other hand, 01110 represents $0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 8 + 4 + 2 = 14$ so $01110 \notin A$. Give a DFA that computes A.

2. Convert the following NFA to a DFA. Simplify if possible.



3. Let $\Sigma = \{a, b\}$, let $A = \{w\colon\ w$ ends in an $a\}$, and let $B = \{w\colon\ w$ has an odd number of b's$\}$. Give a DFA for the language AB and then simplify.

4. Given a language A and a non-negative integer k, we define A^k to be the set of words w obtained by concatenating k words in A. We also define $A^* = \bigcup_{k \geq 0} A^k$; that is, A^* consists of all strings that can be obtained by concatenating zero or more strings in A.

   (a) Let $\Sigma = \{a, b\}$ and let $A = \{w\colon\ w$ starts and ends with different symbols$\}$. Give a simple, plain English description for the language A^*.

   (b) Use your description in part (a) to give a DFA for A^* with at most 6 states.